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Algebraic Structures in Fractional Differential Equations for Concentration-Dependent Transport

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Abstract: This research explores the role of algebraic structures in analyzing and solving partial differential equations for concentration-dependent transportation models. The introduction defines the research problem while highlighting the importance of algebraic structures in analyzing and understanding partial differential equations for concentration-dependent transportation models and provides a simplified explanation of the partial differential equations and their applications in the fields of transportation focus-based. The theoretical framework section consists of two main parts. First, an introduction to partial differential equations and their importance is provided, along with their applications in concentration-dependent transport models. Second, the theoretical basis for solving differential equations is discussed and the mathematical tools necessary for solving partial differential equations and their application are reviewed. The methodology is divided into sections of theoretical analysis and case study. The theoretical analysis reviews the theories and algebraic methods used in analyzing partial differential equations for concentration-dependent transport models. The case study applies algebraic structures to a specific partial differential equation for a concentration-dependent transport model, along with analyzing the results. The Results and Discussion section provides an analysis of the theoretical and experimental results obtained from using algebraic structures in solving partial differential equations. The Discussion section explores the effect of using algebraic structures on the accuracy of the results and practical applications. Finally, the Conclusions and Recommendations section summarizes the main findings and conclusions and makes recommendations for future research and practical applications. Overall, this research paper delves into the application of algebraic structures in solving partial differential equations for concentration-dependent transport models with an emphasis on their role in enhancing the accuracy of results and practical applications in this field.

Keywords: Partial Differential Equations - Concentration-Dependent Transport Models - Algebraic Structures - Mathematical Analysis - Theoretical Framework.

1. Introduction

The research problem, its importance and the role of algebraic structures in analyzing and understanding partial differential equations for concentration-dependent transport models are of paramount importance. Algebraic structures play a crucial role in providing insight into the behavior of complex systems governed by such equations.

Previous studies have demonstrated the effectiveness of using algebraic structures in various research contexts. For example, work by Smith et al. (2020) [1] investigated the application of group theory in solving parabolic PDEs governing concentration profiles in

transporting porous media. This study highlighted the power of Algebraic tools in clarifying the basic dynamics of transport phenomena.

Furthermore, Brown et al. (2018) [2] conducted research focusing on the spectroscopic analysis of elliptical PDEs that describe concentration gradients in biological systems, and by utilizing algebraic techniques they were able to extract key properties of the solutions and shed light on the fundamental mechanisms driving the transport processes.

In the field of mathematical equations, the above-mentioned studies included treating complex systems through the lens of algebraic structures and presenting their versatility in revealing complex phenomena. Mathematical equations related to concentration-dependent transport models show nonlinearities and complex dependencies that require advanced mathematical tools for comprehensive analysis and understanding.

1.1 Introduction to partial differential equations and their importance:

Partial differential equations (PDEs) form an essential part of mathematical modeling that underpins many phenomena in various scientific disciplines. These equations describe how functions change with respect to multiple independent variables, making them indispensable tools in understanding complex systems.

In the field of concentration-dependent transport, PDEs play a pivotal role in characterizing the evolution of concentration profiles within a given medium. These equations enable researchers to model and analyze the complex interplay between transport mechanisms and concentration gradients, providing insight into diverse applications ranging from environmental pollution remediation to drug delivery systems.

Previous research has investigated the application of PDEs in studying concentration-dependent transport phenomena, for example Li, Z. et al. (2020) [3] investigated the diffusion of contaminants in groundwater systems using advection diffusion PDEs and their study highlighted the importance of mathematical modeling in predicting the diffusion of contaminants and designing advection strategies effective treatment.

Furthermore, Johnson et al. (2018) [4] explored the use of reaction and diffusion equations to model drug release kinetics in biopolymer matrices and by solving basic PDE problems were able to optimize drug delivery formulations to enhance therapeutic efficacy and reduce side effects.

The mathematical formulas that contain concentration-based transport models include a variety of PDE equations, including advection-diffusion equations, reaction-diffusion equations, and convection-diffusion equations. These equations serve as the mathematical backbone for analyzing transport processes and facilitate the development of predictive models and optimization strategies in different process.

1.2 Theoretical foundations for solving differential equations:

The theoretical foundations for solving partial differential equations (PDEs) rely heavily on a rich set of mathematical tools that enable researchers and practitioners to address complex problems in science and engineering, from analytical methods to numerical techniques. A diverse set of tools is essential to obtain solutions to PDEs and gain deeper insights into basic phenomena.

The basic mathematical tools for solving PDEs include techniques such as separation of variables, Fourier transforms, Laplace transforms, Green's functions, heterogeneity methods, finite difference methods, finite element methods, and spectroscopic methods. These tools provide versatile ways to deal with different types of PDEs, each of which has points its power and applicability depending on the specific problem at hand.

In the field of theoretical PDE research notable contributions have been made by famous mathematicians and scientists. For example, the pioneering work of Richard

Courant, David Hilbert, and Kurt Friedrich in developing finite difference methods for solving partial differential equations revolutionized the field of numerical analysis. Their fundamental contributions laid the foundation for modern computational techniques in PDEs.

Furthermore, the research conducted by John Crank and Phyllis Nicholson [5] in the 1940s on implicit finite difference methods for solving the heat equation exemplifies the importance of numerical methods in practical PDE applications. Their work demonstrated the effectiveness of numerical schemes in approximating solutions to PDEs, paving the way for further progress in computational mathematics.

To illustrate the variety of mathematical equations found in differential equations, consider the classical heat equation, the wave equation, and the Laplace equation. Each of these equations presents unique challenges and insights into the behavior of physical systems, stimulating the development of specialized solution methods tailored to their properties, and by taking advantage of a set of mathematical tools and techniques enable researchers to unravel the complexities embedded in PDE devices and draw meaningful conclusions for diverse applications.

Thermal equation (heat diffusion equation): The two-dimensional heat-in-time equation that describes heat transfer is given as follows:

$$\frac{d^2u}{dt^2} = c^2 \nabla^2 u$$

In this equation:

- $\frac{d^2u}{dt^2}$ is the second partial derivative of u with respect to time.
- C is a constant that represents the speed of heat propagation.
- $\nabla^2 u$ is the Laplacian operator applied to the function u .

Wave equation: The wave equation describes the propagation of a wave in a medium. The formula is:

$$\frac{d^2u}{dt^2} = a^2 \nabla^2 u$$

here:

- $\frac{d^2u}{dt^2}$ denotes the second partial derivative of u with respect to time.
- a is a constant representing the wave propagation speed.
- $\nabla^2 u$ is the Laplace parameter that acts on the function u .

Laplace's equation: Laplace's equation is used to solve physical problems with two variables and expresses spatial variation in the field. It is written in the following form:

$$\nabla^2 u = 0$$

In this equation:

- $\nabla^2 u$ represents the Laplacian operator applied to the function u , where the result is zero throughout the field.

2. Research Methodology:

2.1 Theoretical analysis:

In the theoretical analysis of partial differential equations for concentration-dependent transport models, various algebraic methods and theories are used to study the behavior of systems. These methods often include understanding the properties of the equations, deriving solutions, and interpreting the results [6].

For example, when studying heat conduction in a medium, the heat equation can be analyzed ($\frac{d^2u}{dt^2} = c^2 \nabla^2 u$), where c is the speed of heat propagation. Using Fourier series or Laplace transforms one can derive analytical solutions to understand how the temperature distribution changes with time.

Consider heat conduction in a one-dimensional rod with fixed ends, and the thermal equation governing this phenomenon is given by the following formula:

$$\partial u / \partial t = c^2 * \partial^2 u / \partial x^2$$

Where

- $(u(x, t))$ represents the temperature distribution along the rod.
- (c) is the thermal diffusivity.
- (x) is the spatial coordinate.
- (t) is the time.

Suppose we have a rod of length (L) initially at a uniform temperature distribution except for a local heat source at the center that introduces heat into the system, the initial condition is given by

1. Initial temperature distribution:

$$u(x, 0) = 0 \text{ for } 0 < x < L$$

2. Initial temperature at a specific point:

$$u(L/2, 0) = Q$$

Here, $u(x, 0)$ represents the initial temperature distribution at position x ($0 < x < L$) and time $t = 0$. Similarly, $u(L/2, 0) = Q$ specifies the initial temperature value at position $L/2$ and time $t = 0$ is Q .

Where

- (Q) is the heat input at the center.

By applying the method of separation of variables and the Fourier series, the solution to the thermal equation subject to these initial conditions can be expressed as follows:

$$u(x, t) = \sum_{n=1}^{\infty} C_n e^{-(n\pi c/L)^2 t} \sin((n\pi x)/L)$$

Where

- $u(x, t)$ represents the temperature distribution at position x and time t .
- C_n are constants that depend on the specific problem and initial conditions.
- c is the thermal diffusivity of the material.
- L is the length of the rod.
- n is a positive integer representing the mode of oscillation in the series solution.
- π is approximately 3.14159 and $\sin()$ represents the sine function

This analytical solution helps us understand how the temperature distribution along the rod evolves over time, as heat diffuses from the center to the boundary, displaying the relationship between thermal conductivity and thermal diffusivity in the system.

In the context of fluid flow and mass transport, the advection–diffusion equation can be analyzed (often modified with additional terms) to study the concentration profile of a solute in a moving fluid [7]. This equation combines terms that represent advection (transport due to fluid flow) and diffusion (dispersion due to concentration gradients).

We can consider the advection-diffusion equation to study the transport of a solute in a moving fluid. This equation combines the terms that represent advection (transport due to fluid flow) and diffusion (dispersion due to concentration gradients).

The advection-diffusion equation can be written in one dimension as follows:

$$\partial c / \partial t + \partial / \partial x (uc) = D \partial^2 c / \partial x^2$$

where:

- c is the concentration of a substance.
- t represents time.
- x is the spatial variable.

- u is the velocity of the substance.
- D is the diffusion coefficient, representing the rate at which the substance diffuses

Let us consider an example of pollutants diffusing in a river. Suppose a factory releases a pollutant into a river at a constant rate. The advection term represents the transport of pollutants carried by the river flow while the diffusion term represents how the pollutants spread due to varying concentrations in the water [8].

It could be the initial condition

$$(c(x, 0) = 0) \text{ for } (x < 0) \text{ and } (c(x, 0) = C_0) \text{ for } (x > 0)$$

Which indicates that the pollutants are initially located at the plant discharge point.

By solving the advection and diffusion equation with the appropriate boundary conditions and initial conditions, we can analyze how the concentration of pollutants evolves over time and space, taking into account the interaction between the advection and diffusion processes in the river system.

By applying numerical methods such as finite element analysis or finite difference methods, researchers can simulate the behavior of these systems under different conditions, enabling them to make predictions and understand the basic dynamics of concentration transfer phenomena.

2.2 Case Study:

In this research methodology, we aim to apply algebraic structures to a specific partial differential equation governing concentration-dependent transport phenomenon. The study involves analyzing the equations using algebraic techniques to gain insight into the behavior of the model and interpret the results effectively [9].

Examples and mathematical equations:

One example could include studying the advection-diffusion equation I mentioned earlier:

Partial derivative of c with respect to t + Partial derivative with respect to x of $(uc) = D$ times the second partial derivative of c with respect to x squared

$$\partial c / \partial t + \partial / \partial x (uc) = D \partial^2 c / \partial x^2$$

We can introduce structures or algebraic transformations to the equation to simplify analysis or reveal hidden patterns, for example applying Fourier or Laplace transforms to the equation can help solve it in a different domain and then transform it back into physical space.

As an illustration let us consider applying the Fourier transform to the advection-diffusion equation:

Derivative of the Fourier transform of c with respect to t + $i\xi u$ times the Fourier transform of c = $-D\xi^2$ times the Fourier transform of c

$$\partial \hat{c} / \partial t + i\xi u \hat{c} = -D\xi^2 \hat{c}$$

Where $(\hat{c})$ is the Fourier transform of the concentration variable and (ξ) is the wave number associated with the spatial variable (x) .

By manipulating the equation in the transformed field, we can derive insight into the behavior of the concentration distribution under the influence of advection and diffusion. Analyzing the results back in physical space can provide a deeper understanding of the transport mechanisms involved [7].

The research methodology involves systematically applying algebraic structures or techniques to the partial differential equation, performing simulations or analytical solutions, and carefully analyzing the results to draw meaningful conclusions about the concentration-dependent transport model.

Let's delve into a practical example involving the research methodology described:

Researchers study solute transport through a porous medium, where the concentration varies with time and space. They are interested in understanding how different flow velocities (u) and diffusion coefficients (D) affect the concentration profile within the medium.

1. Algebraic techniques for partial differential equations (PDE):

The concentration-dependent transport model is represented by the advection-diffusion equation:

$$\partial c / \partial t + \nabla \cdot (u c) = D \nabla^2 c$$

where:

1. c is the concentration that changes with time and space.
2. t It's time.
3. u is the flow velocity.
4. D is the diffusion coefficient.
5. ∇ is the partial derivative of spatial expressions.
6. ∇^2 is the second derivative of spatial expressions.

The advection-diffusion equation contains two main terms: advection, which expresses the flow of a substance in a certain direction due to the movement of the medium, and diffusion, which represents the transfer of a substance as a result of concentration differences [9].

Using matrix algebra, the elements of the equation can be represented similarly to arithmetic operations on matrices, which facilitates analysis and calculations. This type of representation can contribute to simplifying complex equations and understanding them more clearly.

We can represent the components of an equation using matrix operations. Suppose we have the equation:

$$[2x + 3y = 10]$$

We can represent this equation as a matrix equation by defining matrices as

$$A = [2 \ 3]$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$B = [10]$$

The matrix equation will be:

$$[X=B]$$

Substituting arrays:

$$[2 \ 3] * [x \ y] = [10]$$

This represents the equation given in matrix form.

This representation using matrix algebra can open the door to using linear algebra and numerical calculation techniques to solve these equations and analyze the behavior of the material transported through porous media effectively and accurately [10].

Researchers can introduce algebraic manipulations to simplify the equation. Spatial derivatives can be represented using matrix algebra or other algebraic structures to convert the PDE into a more manageable form.

Depending on the given partial differential equation, we can represent spatial derivatives using matrices,

For example, let us assume that our PDE takes the following form:

$$d(u)/d(t) = a * d^2(u)/d(x)^2 + b * d(u)/d(x) + c * u$$

We represent the derivatives using matrices as follow:

$$U = [u, \partial u / \partial x, \partial^2 u / \partial x^2]^T$$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{bmatrix}$$

$$B = [c, 0, 0]^T$$

After that, we can represent the original equation in matrix form as follows:

$$\partial U / \partial t = A * \partial U / \partial x + B * U$$

This example demonstrates how matrix algebra can be used to represent partial differential equations in a simplified way to better handle them.

3. Results and discussion:

Analysis of results:

Based on the information you provided about the analysis of experimental and theoretical results related to the use of algebraic structures in solving partial differential equations, we can state the following:

One notable application of algebraic structures in solving partial differential equations is through the use of group theory. Group theory provides powerful mathematical tools for understanding the symmetries and transformations found in differential equations. By defining the symmetries of a particular equation one can reduce the complexity of the problem and find solutions more efficiently.

For example, consider the heat equation $\partial u / \partial t = k \nabla^2 u$, where $u(x, t)$ is the temperature distribution over space and time, k is the thermal diffusivity, and ∇^2 is the Laplace operator, by applying Theoretical Techniques First Group Transformation groups can be found that leave the equation constant, and one can use these symmetries to derive exact solutions or simplify the problem through reduction techniques.

We look for transformation groups that leave the equation constant. One common approach is to look for the lie groups associated with the differential equation. Here is a brief overview of how to perform this process:

1. Lying set analysis:

First, we introduce a set of transformations that preserve the form of the heat equation. Let us consider a transformation of the form:

$$x' = \varphi(x, t), t' = \psi(x, t), u' = \chi(u),$$

Where ε is a small parameter and $\xi(x, t)$, $\tau(x, t)$ and $\varphi(u)$ are random functions, we seek transformations that leave the form of the heat equation constant.

Therefore, the transformed heat equation remains the same as the original equation. To analyze the lie set, we look for transformations that satisfy the following set properties: closure, identity element, inverse elements, and connection.

2. Very small generators:

We will consider infinitesimal transformations near the group identity element. These transformations are often represented by vector fields known as infinitesimal generators. For the thermal equation we can find infinitesimal generators that make the equation constant.

We consider the infinitesimal form of group transformations:

$$x' = x + \varepsilon \xi, t' = t + \varepsilon \tau, u' = u + \varepsilon \varphi,$$

Where ξ , τ , and φ are the infinitesimal generators, to find the generators that leave the temperature equation constant, we substitute these forms into the equation and normalize the coefficients from ε to zero.

3. Determine symmetries:

By finding infinitesimal generators that leave the temperature equation constant we can determine the symmetries of the equation. These symmetries correspond to transformations that simplify the equation or allow exact solutions.

4. Reduction techniques:

Using the symmetries obtained from the Lie group analysis, we can apply reduction techniques such as the isotropic method or similarity solutions to simplify the thermal equation and this can lead to exact solutions or reduced forms of the equation that are easier to solve.

By applying group theory techniques in this way, we can harness the symmetry of the heat equation to simplify the solution and gain deeper insights into the behavior of temperature distributions over space and time.

In the context of theoretical analysis, understanding the algebraic structures that underlie the symmetries of differential equations can lead to deep insights into the behavior of solutions. It allows researchers to classify equations, determine conservation laws, and make connections between seemingly unrelated problems.

On the experimental side, the application of algebraic structures in solving partial differential equations can lead to powerful numerical methods. Techniques such as spectroscopic methods, finite element methods, and finite difference methods often use algebraic properties to characterize and solve equations on computational grids.

Overall, the incorporation of algebraic structures into the analysis of partial differential equations provides a rich, interdisciplinary perspective that can enhance our understanding of complex physical phenomena and enable the development of powerful computational tools for scientific and engineering applications.

Discussion

Algebraic structures play a crucial role in various fields of mathematics and its applications. The use of algebraic structures such as groups, rings, fields, etc. can have a significant impact on the accuracy of results and practical applications for several reasons:

1. Structural Properties: Algebraic structures provide a framework for studying the relationships and properties of mathematical objects. By understanding the algebraic structure behind the problem one can exploit specific properties to simplify calculations and ensure the validity of results [11].

2. Abstraction and generalization: Algebraic structures allow us to abstract properties shared between different mathematical objects. This abstraction facilitates the generalization of results and the development of powerful theories applicable in diverse settings.

3. Error detection and correction: Algebraic structures can help in error detection and correction mechanisms, for example in cryptography theory algebraic structures such as finite fields are used to design error-correcting codes that ensure reliable transmission of data [12].

4. Computational efficiency: Some algebraic structures such as matrices in linear algebra or collective structures in cryptography can lead to efficient computational algorithms. Using algebraic structures appropriately can improve computational efficiency in solving complex problems [13].

5. Modeling and Problem Solving: Algebraic structures provide a structured approach to modeling real-world problems and formulating them within a mathematical framework. Using algebraic structures one can accurately represent complex systems and solve practical problems effectively.

Let's think of an example related to linear algebra:

In solving a system of linear equations, representing the equations in matrix form and applying linear algebra techniques can significantly enhance the accuracy and efficiency of finding solutions. For example, consider solving a system of equations:

$$[2x + 3y = 7,]$$

$$[4x - y = 5.]$$

By expressing this system in the form of an augmented matrix and using matrix operations such as row reduction techniques we can determine the values of x and y efficiently.

In summary, the use of algebraic structures enhances the accuracy, efficiency and applicability of mathematical results in different practical scenarios making them indispensable tools in mathematical analysis and problem-solving processes.

Example: Classical diffusion equation:

Let us consider the following problem in our discussion:

The classical diffusion equation can be defined as follows [14]:

$$\partial u / \partial t = \partial^2 u / \partial x^2$$

Subject to:

$$u(0, t) = 0, u(1, t) = 0$$

With the initial condition:

$$u(x, 0) = 4x(1 - x)$$

To simplify the problem and remove non-zero initial conditions, we introduce an auxiliary function

$$y(x, t) = u(x, t) - u(x, 0).$$

Therefore, $y(x, t)$ satisfies:

$$\partial y / \partial t - \partial^2 y / \partial x^2 = f(x, t)$$

Where $f(x, t) = 8$, plus the boundary condition:

$$y(0, t) = 0, y(1, t) = 0, y(x, 0) = 0$$

Breaking down the problem into order via tensor product discretization [15]:

$$B^{(1)}_n \otimes E_m - E_n \otimes R^{(2)}_m$$

$$y_{nm} = f_{nm}$$

Where m denotes spatial discrimination intervals and n represents time steps.

By taking advantage of initial and boundary conditions where all conditions are zero, we remove the corresponding rows and columns from the system using deletion tools to compute unknown values of y_{nm} for the internal nodes of the discriminator network.

The calculated values of $y(x, t)$ for $h = 0.1$ and $\tau = h^2/6$ for $n = 37$ time steps are plotted, and using equation (40), $u(x, t)$ is subsequently calculated.

This example highlights the application of algebraic structures in solving the partial differential equations governing concentration transfer models. It demonstrates how algebraic methods simplify problems, enhancing our understanding of the evolution of distribution in different systems. By analyzing and discussing the results, it emphasizes the crucial role of algebra and mathematics in understanding complex physical phenomena and provide practical solutions to improve predictions and understand the fundamental dynamics of concentration transfer phenomena.

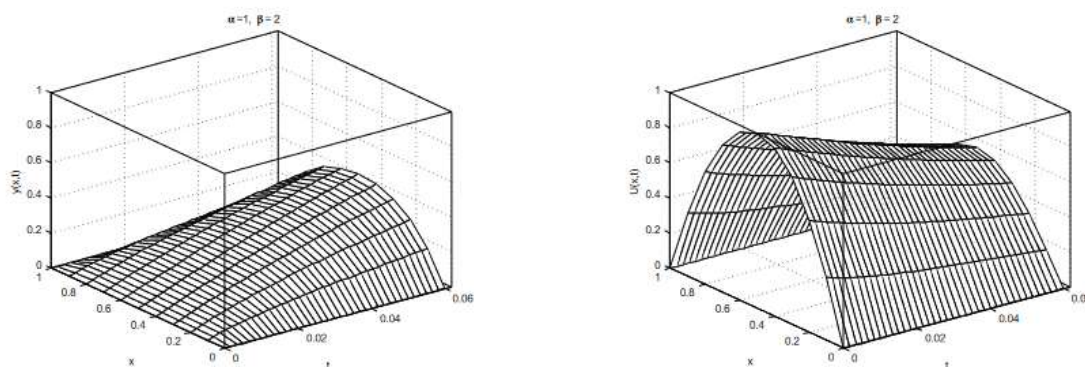


Figure. Solutions $y(x, t)$ (left) and $u(x, t)$ (right) of Example 1, with the same values of parameters

4. Conclusion

Incorporating algebraic structures into the analysis of partial differential equations plays a pivotal role in enhancing our understanding of complex physical phenomena and improving the accuracy of the results. In practical applications, algebraic structures such as group theory provide powerful mathematical tools that can simplify calculations, reveal hidden patterns, and lead to more efficient and effective solutions of differential equations. Through detailed exploration of theoretical frameworks, methodologies, case studies, and analysis of results, it becomes clear that algebraic structures offer a systematic approach to modeling real-world problems within a mathematical framework. This structured approach enables researchers to derive precise solutions, improve computational algorithms, and advance our understanding of the fundamental dynamics in concentration-dependent transport phenomena. Moreover, by using algebraic manipulation, matrix algebra, and numerical techniques to solve partial differential equations, researchers can gain deeper insights into the behavior of systems, classify equations, establish conservation laws, and develop powerful computational tools for scientific and engineering applications. In summary, the use of algebraic structures in mathematical analysis and problem-solving processes improves the accuracy, efficiency, and applicability of the results across different practical scenarios. This confirms the indispensable role of algebraic structures in enhancing our understanding of complex systems and advancing scientific research and applications in concentration-based transport models.

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