



Article

Complex Curvature Tensor of Generalized Veissmann Manifold

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Abstract: In this post, we will look at the geometrically conharmonic tensor. A smooth curvature tensor connected to a generalized Veissman Grey manifold. We make use of the GV^* -manifold flattening property. It was demonstrated in the paper that $[(GV^*)]_1 \otimes [(GV^*)]_2 \otimes [(GV^*)]_3$ is the generalized of the Veissman Grey manifold. The fundamental goal of this paper is to study some architectural aspects of the Veissman Grey manifold, which is distinguished by flattened round curvature. Important requirements for the Veissman Grey manifold, locally conformal, Kehler, and margins are established, and new connections between the two are discovered, using the asymmetry property of the circle pair. Additionally, we looked into the transverse curvature, this provided to us an abundance of knowledge on the Riemann the study of geometry, a region crucial to the study of differential geometry. Circular transformations were important in Rumanian structural changes to maintain consistency of these harmonic structures.

Keywords: Veissman Grey manifold, Rumanian structural, Complex Curvature.

1. Introduction

The surface of the previously stated conharmonic tensor is studied in Riemann space. Gray [1] discovered three types of nearly manifolds, each of which has a Riemann representation tensor. The groups are Rakes [2] studied the locally kehler of class R_3 in 2008, assuming the Lie algebraic form's component is genuine. R_1 , R_2 , & R_3 can be used to describe the intricate concircular matrices of specific kinds of nearly hermition, which Saadan explored in 2011 [3]. To correctly determine its requirements and qualities, the topic was divided into a number of elements. As a result, a significant issue arose: the organization of the different kinds of Hermitical manifolds according to particular properties [4]. They found the underlying equation in the adjoining G-structure space [5]. Kerichenko and Eshova investigated the topological characteristic of class $W_1 \oplus W_4$ in 1996 [6]. The graph H of give C_i meets the requirement. For all

$i = 1, 2, 3$, if the value is part of a subset of A-structures, immediately $\theta \cap C_i = \emptyset$, $i = 1, 2, 3$ [7]. Among these is a Russian physicist named Kerichenko [8], Conharmonic with these curvature tensors is feasible. The formulae of a valid VG-manifold are determined by Ignatochkina using structured and unstructured teasing [9]. This fundamental idea is known as the adjoint VG-structure space [10]. Through these processes, we were able to gain a better understanding of the VG-manifold class's projected field [11]. He applied VG-space to studies the Hermition manifold, that isn't reliant on a manifold, but on the fundamental that applies to soluton [12]. Their discovered the equation of the composition of the adjacent A-structure [13]. Specifically, it was demonstrated by Veissman Grey that the theomorphic conharmonic (CGV(n)) bending tensors are uniform at all points

provided that specific requirements are satisfied by the components within the neighboring C-structure. [14]. Many academics have studied the nearly Hermitically manifold [15]. In 2015, Al-Hwez studied the internal makeup of the canharmonic curving structure of the Kehler and nearly Kehler [16]. When $j_1 = m(a_e)$ represents the frame's the same, this foundation has been referred to as a nearly structure premise or almost basis. This is commonly referred to as an A-frame [17].

2. Preliminaries

$X(M)$ must have smooth in texture. M denotes the a vector space component. Let $C^\infty(M)$ denote a function group on M . The parallel space $T_p(M)$, $(J_p)^2 = \text{id}$ $g = \langle \dots \rangle$ Matrix Riemannian on G [18]. The base is $[e_1 \dots, e_n \dots | e_1 \dots | e_n]$. The foundation $[i_1, \dots, i_n, \dots, \bar{i}_1, \dots, \bar{i}_n]$. Is used to build the latest, this is denoted by $\{J, g\}$. With $i_a = \sigma(e_a)$ and $\bar{\sigma} = e_a$. The framework has been referred to as nearly structure. The producer's associated $[P, \dots, i_1, \dots, i_n, \dots, \bar{i}_1, \dots, \bar{i}_n]$ is an A-frame.

The measurements in the range of $1, \dots, 2n$ are u, g, l , and P . The figures $1, 2, \dots, k$ will be used. With $\hat{a} = a + n$, use the indicators $[i_{\hat{1}} = \bar{i}_1, \dots, i_{\hat{n}} = \bar{i}_n]$. You may write frame $[p, i_1, \dots, i_n, \dots, \bar{i}_1, \dots, \bar{i}_n]$ using that form. The parts of the matrix of the complicated structure with environment adjoined the following Q-structure kinds:

$$(\langle JX, JY \rangle J_i^j) = \begin{pmatrix} \sqrt{-1} I_n & 0 \\ 0 & -\sqrt{-1} I_n \end{pmatrix}, (g_i^j) = \begin{pmatrix} 0 & I_n \\ I_n & 0 \end{pmatrix} \quad (1)$$

Which is the group system [19].

Definition 2.1 [20]

According to Banner's definition of a virtually Herniation, the Veissman unit satisfies these requirements: $B^{abc} = -B^{bac}$, $B_c^{abc} = \alpha^{[a} \delta_c^{b]}$.

Theorem 2.2[21]

The compilations on Viessman-grey equations for structure encompass the following kinds of types:

$$\begin{aligned} \text{i)} & d\omega^a = \omega_b^a \wedge \omega^b + B_c^{ab} \omega^c \wedge \omega_b + B^{abc} \omega_b \wedge \omega_c; \\ \text{ii)} & d\omega_a = -\omega_a^b \wedge \omega_b + B_{ab}^c \omega_c \wedge \omega^b + \omega_{abc} \omega^b \wedge \omega_c; \\ \text{iii)} & d\omega_b^a \omega_c^a \wedge \omega_b^c + (2B^{adh} B_{hbc} + A_{bc}^{ad}) \omega^c \wedge \omega_d + (B^{ah} {}_{[c} B_{d]bh} + A_{bcd}^a) \omega^c \wedge \omega^d + \\ & (B_{bh} {}^{[c} B^{d]ah} + A_b^{acd}) \omega_c \wedge \omega_d; \end{aligned}$$

The metric g has mixed form components ω^i and Riemann connation elements ω_j^i .

Detention 2.3

If $GT_{\hat{a}\hat{b}\hat{c}\hat{d}}$ of conharmonic tensor T equals zero, the component of a Viessman manifold is called a conharmonic Para Kehler manifold.

Theorem 2.4[22]

The generalized Riemann product in the GV^* -manifold in the adjoined A-field consists of three main components:

$$\begin{aligned} 1) GT_{\hat{a}\hat{b}\hat{c}\hat{d}} &= \left\{ -A_{bd}^{ac} + B^{ah} {}_b B_{hd}^c + \frac{1}{2} \alpha_{[b}^{[a} \delta_{d]}^c \right\} \\ & \left\{ A_{bc}^{ad} - B^{ah} {}_c B_{hb}^d + \frac{1}{2} \alpha_{[b}^{[a} \delta_{c]}^d \right\} \end{aligned} \quad 2) GT_{\hat{a}\hat{b}\hat{c}\hat{d}} =$$

Detention 2.5[23]

The kind of tensor that describes the shape of a circle within an C-manifold is $(4,0)$. And met the condition $e^{-2f} C(K, L, N, M) = C(K, L, N, M)$, which is described as the shape:

$$GV^*_{abcd} = T_{abcd} - \frac{k}{4n(n+1)} [g_{ad}g_{bc} - g_{ac}g_{bd}] - \frac{k}{4n(n+1)} [\Omega_{ad}'\Omega_{bc} - \Omega_{ac}'\Omega_{bd} - 2\Omega_{ab}'\Omega_{cd}]$$

Theorem 2.6

The components Complex tensor of GV^* –manifold in the adjoined C- space are provided in the kinds of documents listed below:

Proof:

1) Let $i = a, j = b, h = c$ and $l = d$

$$GV^*_{abcd} = T_{abcd} - \frac{k}{4n(n+1)} [g_{ad}g_{bc} - g_{ac}g_{bd}] - \frac{k}{4n(n+1)} [\Omega_{ad}'\Omega_{bc} - \Omega_{ac}'\Omega_{bd} - 2\Omega_{ab}'\Omega_{cd}] = 0$$

2) Let $i = \hat{a}, j = b, h = c$ and $l = d$

$$GV^*_{\hat{a}bcd} = T_{\hat{a}bcd} - \frac{k}{4n(n+1)} [g_{\hat{a}d}g_{bc} - g_{\hat{a}c}g_{bd}] - \frac{k}{4n(n+1)} [\Omega_{\hat{a}d}'\Omega_{bc} - \Omega_{\hat{a}c}'\Omega_{bd} - 2\Omega_{\hat{a}b}'\Omega_{cd}] = 0$$

3) Let $i = a, j = \hat{b}, h = c$ and $l = d$ $GV^*_{a\hat{b}cd} = T_{a\hat{b}cd} - \frac{k}{4n(n+1)} [g_{ad}g_{\hat{b}c} - g_{ac}g_{\hat{b}d}] - \frac{k}{4n(n+1)} [\Omega_{ad}'\Omega_{\hat{b}c} - \Omega_{ac}'\Omega_{\hat{b}d} - 2\Omega_{a\hat{b}}'\Omega_{cd}] = 0$

4) Let $i = a, j = b, h = \hat{c}$ and $l = d$

$$GV^*_{ab\hat{c}d} = T_{ab\hat{c}d} - \frac{k}{4n(n+1)} [g_{ad}g_{b\hat{c}} - g_{a\hat{c}}g_{bd}] - \frac{k}{4n(n+1)} [\Omega_{ad}'\Omega_{b\hat{c}} - \Omega_{a\hat{c}}'\Omega_{bd} - 2\Omega_{ab}'\Omega_{\hat{c}d}] = 0$$

5) Let $i = a, j = b, h = c$ and $l = \hat{d}$

$$GV^*_{abc\hat{d}} = T_{abc\hat{d}} - \frac{k}{4n(n+1)} [g_{a\hat{d}}g_{bc} - g_{ac}g_{b\hat{d}}] - \frac{k}{4n(n+1)} [\Omega_{a\hat{d}}'\Omega_{bc} - \Omega_{ac}'\Omega_{b\hat{d}} - 2\Omega_{ab}'\Omega_{c\hat{d}}] = 0$$

6) Let $i = \hat{a}, j = \hat{b}, h = c$ and $l = d$

$$GV^*_{\hat{a}\hat{b}cd} = T_{\hat{a}\hat{b}cd} - \frac{k}{4n(n+1)} [g_{\hat{a}d}g_{\hat{b}c} - g_{\hat{a}c}g_{\hat{b}d}] - \frac{k}{4n(n+1)} [\Omega_{\hat{a}d}'\Omega_{\hat{b}c} - \Omega_{\hat{a}c}'\Omega_{\hat{b}d} - 2\Omega_{\hat{a}\hat{b}}'\Omega_{cd}] = 0$$

7) Let $i = \hat{a}, j = b, h = \hat{c}$ and $l = d$

$$GV^*_{\hat{a}b\hat{c}d} = T_{\hat{a}b\hat{c}d} - \frac{k}{4n(n+1)} [g_{\hat{a}d}g_{b\hat{c}} - g_{\hat{a}\hat{c}}g_{bd}] - \frac{k}{4n(n+1)} [\Omega_{\hat{a}d}'\Omega_{b\hat{c}} - \Omega_{\hat{a}\hat{c}}'\Omega_{bd} - 2\Omega_{\hat{a}b}'\Omega_{\hat{c}d}]$$

$$GV^*_{\hat{a}b\hat{c}d} = -A_{bd}^{ac} + B_{bd}^{ah}B_{hd}^c + \frac{1}{2}\alpha_{[b}^{[a}\delta_{d]}^c] - \frac{k}{4n(n+1)} [\delta_d^a\delta_b^c] - \frac{k}{4n(n+1)} [\delta_d^a\delta_b^c + 2\delta_b^a\delta_d^c]$$

$$GV^*_{\hat{a}b\hat{c}d} = -A_{bd}^{ac} + B_{bd}^{ah}B_{hd}^c + \frac{1}{2}\alpha_{[b}^{[a}\delta_{d]}^c] - \frac{k}{4n(n+1)} [2\delta_d^a\delta_b^c + 2\delta_b^a\delta_d^c]$$

$$GV^*_{\hat{a}b\hat{c}d} = -A_{bd}^{ac} + B_{bd}^{ah}B_{hd}^c + \frac{1}{2}\alpha_{[b}^{[a}\delta_{d]}^c] - \frac{k}{2n(n+1)} \tilde{\delta}_{bd}^{ac}$$

8) Let $i = \hat{a}, j = b, h = c$ and $l = \hat{d}$

$$GV^*_{\hat{a}bc\hat{d}} = T_{\hat{a}bc\hat{d}} - \frac{k}{4n(n+1)} [g_{\hat{a}d}g_{bc} - g_{\hat{a}c}g_{b\hat{d}}] - \frac{k}{4n(n+1)} [\Omega_{\hat{a}d}'\Omega_{bc} - \Omega_{\hat{a}c}'\Omega_{b\hat{d}} - 2\Omega_{\hat{a}b}'\Omega_{c\hat{d}}]$$

$$GV^*_{\hat{a}bc\hat{d}} = A_{bc}^{ad} - B_{bc}^{ah}B_{hb}^d + \frac{1}{2}\alpha_{[b}^{[a}\delta_{c]}^d] + \frac{k}{4n(n+1)} [\delta_c^a\delta_b^d] + \frac{k}{4n(n+1)} [\delta_c^a\delta_b^d + 2\delta_c^a\delta_b^d]$$

$$GV^*_{\hat{a}bc\hat{d}} = A_{bc}^{ad} - B^{ah}_c B_{hb}^d + \frac{1}{2} \alpha_{[b}^{[a} \delta_{c]}^{d]} - \frac{k}{4n(n+1)} [2\delta_c^a \delta_b^d + 2\delta_c^d \delta_b^a]$$

$$GV^*_{\hat{a}bc\hat{d}} = A_{bc}^{ad} - B^{ah}_c B_{hb}^d + \frac{1}{2} \alpha_{[b}^{[a} \delta_{c]}^{d]} - \frac{k}{2n(n+1)} \delta_{bc}^{ad}$$

By using the properties of complex tensor we have

$$GV^*_{\hat{a}b\hat{c}d} = GV^*_{\hat{a}bc\hat{d}} \text{ as follows}$$

$$GV^*_{\hat{a}b\hat{c}d} = -GV^*_{\hat{a}bc\hat{d}}$$

Theorem 2.7

If M is a GV^* -manifold of any the flat complex tensor, then M is of zero complex general scalars.

Proof:

By using (Theorem 2.6) we get:

$$GV^*_{\hat{a}bc\hat{d}} = A_{bc}^{ad} - B^{ah}_c B_{hb}^d + \frac{1}{2} \alpha_{[b}^{[a} \delta_{c]}^{d]} - \frac{k}{2n(n+1)} \delta_{bc}^{ad}$$

Since that GV^* -manifold is a flat complex tensor.

Then $GV^*_{\hat{a}b\hat{c}d} = 0$

$$A_{bc}^{ad} - B^{ah}_c B_{hb}^d + \frac{1}{2} \alpha_{[b}^{[a} \delta_{c]}^{d]} - \frac{k}{2n(n+1)} \delta_{bc}^{ad} = 0$$

When the index (a, b) and (d, c) are contracted, we get

$$B^{ah}_b B_{hd}^c = 0$$

$$\text{There for } -A_{bd}^{ac} + \frac{1}{2} \alpha_{[b}^{[a} \delta_{d]}^{c]} - \frac{k}{2n(n+1)} \delta_{bd}^{ac} = 0$$

When the index (a, b) & (d, c) are contracted, we get

$$-A_{bd}^{ac} - \frac{k}{2n(n+1)} = 0$$

$$\text{Hence } \frac{k}{2n(n+1)} \delta_{bd}^{ac} = 0$$

Ant symmetrizing by the index (a, c) , it follows

$$k = 0$$

Hence M is thus a zero-general scalars.

Theorem 2.8

Let M be the $-$ manifold of a Complex tensor, Then M is NK-manifold.

Proof:

Suppose M is the GV^* $-$ manifold of a generalized Complex tensor.

Using (Theorem 2.6) we obtain

$$GV^*_{\hat{a}bc\hat{d}} = -A_{bc}^{ad} + B^{ah}_c B_{hb}^d - \frac{1}{2} \alpha_{[b}^{[a} \delta_{c]}^{d]} + \frac{k}{2n(n+1)} \delta_{bc}^{ad}$$

Using the index (a, d) to symmetrize and antisymmetrize, we get

$$B^{ah}_b B_{hd}^c = 0$$

When the index (a, b) & (d, c) are contracted, we get

$$B^{ah}_b B_{ha}^b = 0 \Leftrightarrow B^{ah}_b \bar{B}^{ah}_b = 0 \Leftrightarrow \sum_{a,b,h} \|B^{ah}_b\|^2 = 0 \Leftrightarrow B^{ah}_b = 0$$

Therefore, M is NK-manifold.

$$\text{GV}^*_{bcd}{}^a = \text{GV}^*_{b\hat{c}\hat{d}}{}^a = \text{GV}^*_{\hat{b}\hat{c}\hat{d}}{}^a = 0.$$

3. Conclusions

Researchers identified analogous groupings and their relationship to the basis of a hierarchy manifold by examining the harmonics of the Viesman curve. It was also possible to figure out the normal curvature sources for each unit and how they related to algebraic ideas. shape and the well-known curve were found. For example, this connection shows the shape of the Remain, which in turn shows some frames that let us study the needs structures of all kinds of Hermitic manifolds. Finally to prove that $(VG)_1 \subset (VG)_2 \subset (VG)_3$.

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