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Article

Mathematical Optimization Framework for Personalized Hypertension Dietary Planning Using Multi-Objective Fuzzy Goal Programming Under Nutritional Uncertainty

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Abstract: Blood pressure is one of the most common chronic diseases, and a leading cause of cardiovascular disease, stroke, renal failure, and early death. Although nutritional intervention is an integral part of BP control in non-pharmacologic ways, designing affordable, nutritionally adequate diets is a complex decision-making process. Typical linear programming models in the classical approach assume that food composition, nutritional requirements and nutritional targets are known quantities. Food preparation, market fluctuations, person-to-person health differences, and uncertainty in dietary recommendations, however, can influence these quantities in practice. In this paper, a multi-objective fuzzy goal programming model is proposed to solve the personalized hypertension diet planning problem in the context of nutritional uncertainty. The suggested model not only optimizes the dietary cost but also maximizes the dietary quality and cuts down on sodium intake without compromising the nutritional requirements for calcium, potassium, magnesium, protein, dietary fiber and vitamin E. Imprecise nutritional satisfaction is described by fuzzy membership functions and goal programming is used to convert the conflicting objectives into a single compromise model. Theoretical properties of the model are examined such as feasibility, convexity of the feasible region and continuity of the weighted deviation objective. The representative food data are used in a numerical experiment to demonstrate the behavior of the models and then analysis of sensitivity is undertaken to the price variation, higher nutritional requirement, limited availability and restriction on sodium. The outcomes suggest that the proposed fuzzy system is more flexible and explains the decisions better than the deterministic cost minimisation approach. The model is suitable for mathematical and research purposes only and should be used in conjunction with professional nutritional and medical advice.

Keywords: Hypertension, Fuzzy goal programming, Multi-objective optimization, Personalized nutrition, Dietary planning, Nutritional uncertainty, Healthcare decision support

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1. Introduction

Hypertension is a significant public health problem as it is associated with cardiovascular diseases, stroke, kidney disease and other chronic conditions. While medication is often needed, dietary management is also an important aspect of prevention and maintenance of normal blood pressure. High potassium, calcium, magnesium and dietary fiber intake has been consistently linked to lower blood pressure. One of the better-known diet patterns that has been established for this is the Dietary Approaches to Stop Hypertension (DASH) diet.

Not only is dietary planning a nutritional problem, but it is also an optimization problem. Available food items must be chosen in appropriate amounts to meet several nutrient requirements by a patient, nutritionist or health care decision maker. Meanwhile, the total daily cost should be affordable, especially in communities where an individual's economic status can make unhealthful food choices. These requirements inevitably result in a very limited allocation problem where food amounts are decision variables, nutrition requirements are constraints and the objective functions are cost, quality and sodium content.

The diet problem, for which many of the nutrient-cost relations are linear, is one problem that has been solved successfully by classical linear programming. But in the case of deterministic linear programming, it is generally assumed that the nutrient coefficients of foods, the dietary requirements, and the costs of the foods per unit quantity are known with certainty. This is not always a feasible approach. The nutrient composition may vary depending on the mode of preparation, food source, storage conditions and measurement error. Furthermore, the number of individuals with health needs may not be accurately described by any single number. In these situations, fuzzy optimization is a suitable mathematical model for the representation of uncertainty and partial satisfaction.

In the present study, a multi-objective fuzzy goal programming approach to the problem of personalized dietary planning for hypertension is developed. But the proposed model takes into account three objectives at once: reducing the total dietary cost, improving the quality of the diet, and reducing sodium intake. The fuzzy membership functions are used for handling the nutritional requirements, for allowing mathematical representation of partial satisfaction and uncertainty. This is more flexible than a single objective deterministic model.

The main contributions of this paper are:

1. A multi-objective dietary optimization framework is developed for hypertension management.
2. Fuzzy membership functions are introduced to represent uncertainty in nutritional satisfaction.
3. Cost, dietary quality, and sodium intake are incorporated simultaneously through goal programming.
4. Theoretical properties of the model are established, including feasibility, convexity, and continuity.
5. A numerical experiment and sensitivity analysis are presented to demonstrate the practical behavior of the framework.

The rest of this paper is organized as follows. A literature review is provided in section 2. The mathematical formulation is given in Section 3. Fuzzy dietary modeling is introduced in Section 4. In Section 5, the fuzzy goal programming model is developed. Theoretical analysis is presented in Section 6. Computational experiments and sensitivity analysis are presented in sections 7 and 8. A comparative study is presented in Section 9 and the result of the study and conclusions are presented in Sections 10 and 11.

Linear programming is one of the important techniques of OR and widely used in area of resource allocation, production planning, transportation, diet formulation, healthcare decision problems etc [1]. The classical diet problem is a classical application where a set of food items is chosen for meeting nutritional requirements at minimum cost. The problem is still relevant due to the constant changes in the prices of food, health guidelines and the needs of the population.

Optimization of diet for hypertension has become a growing focus of interest. The DASH dietary pattern has been extensively promoted to control blood pressure, due to its high intake of fruits, vegetables, low-fat dairy products, whole grain foods, and minerals, including potassium, calcium and magnesium [2]. There are several studies that have defined diet planning as a linear programming or goal programming problem [3]. These strategies demonstrate that mathematical programming can be used to help contribute to nutritional adequacy and economic affordability.

Recent research has focused on personalized nutrition and computational decision-support systems. Multi-objective recommendation systems and health-aware models have been suggested to address the nutrition, preference, cost, and disease-specific restrictions [4]. These studies reveal that nutrition planning has come to be viewed more as a quantitative and data-based decision problem than simply a list of food recommendations.

Fuzzy set theory has been found useful in the management of imprecise information [5]. In actual dietary planning nutrient requirements can be unknown or variable, and foods may also have variable composition. Several decision making applications have been encountered in which the goals and constraints have not been strictly defined and have been handled by fuzzy programming and fuzzy goal programming [6]. This is an important consideration in nutrition planning because most people are not satisfied or dissatisfied with a food plan; instead they are more likely to be satisfied or dissatisfied over the course of time.

Optimization techniques are also very widely used in engineering reliability allocation and constrained resource allocation. Sulaiman and his colleagues used mathematical and metaheuristic optimization to solve reliability allocation, cost optimization and network systems problems [7]. While reliability allocation and diet planning may sound like two different application areas, they actually have a common mathematical form: limited resources are needed to be allocated under multiple constraints to achieve the optimum system level performance. The close relationship between the two motivates the extension of advanced optimization thinking from engineering system to a healthcare dietary planning problem [8].

The present paper is different from deterministic diet modelling because it incorporates multi-objective optimization, fuzzy membership functions and goal programming in a single solution approach to the problem of dietary planning for hypertension. This combination adds flexibility, while giving a more realistic representation of uncertainty in application of nutrition decisions.

2. Materials and Methods

Let $x_j \geq 0$, $j = 1,2, \ldots, n$, denote the quantity of food item j included in a daily dietary plan. The decision vector is

$$X = (x_1, x_2, \ldots, x_n)^T. \tag{1}$$

Let c_j be the unit cost of food item j, q_j be its dietary quality score, and s_j be its sodium content. The proposed model includes three main objectives.

The first objective minimizes the total dietary cost:

$$Z_1(X) = \sum_{j=1}^{n} c_j\, x_j. \tag{2}$$

The second objective maximizes the total dietary quality:

$$Z_2(X) = \sum_{j=1}^{n} q_j\, x_j. \tag{3}$$

The third objective minimizes sodium intake:

$$Z_3(X) = \sum_{j=1}^{n} s_j\, x_j. \tag{4}$$

Let a_{ij} denote the amount of nutrient i contained in one unit of food item j, and let b_i denote the recommended minimum requirement for nutrient i, where $i = 1,2, \ldots, m$. The nutritional constraints are

$$\sum_{j=1}^{n} a_{ij}\, x_j \geq b_i, \quad i = 1,2, \ldots, m. \tag{5}$$

Food availability and practical consumption limits can be represented as

$$L_j \leq x_j \leq U_j, \quad j = 1,2, \ldots, n, \tag{6}$$

where L_j and U_j are lower and upper bounds, respectively. These bounds prevent unrealistic solutions such as excessive consumption of a single food item.

The feasible dietary region is defined by

$$\Omega = \{X \in \mathbb{R}^n : AX \geq B,\ L \leq X \leq U,\ X \geq 0\}. \tag{7}$$

Deterministic modeling regards every nutritional requirement as being pre-determined. Nevertheless, in real-life applications, both nutritional requirements and defined values of food components can be highly uncertain. Fuzzy modeling gives us an appropriate means of expressing that uncertainty through membership functions.

For nutrient i, let L_i be the minimum unacceptable level and U_i be the fully satisfactory level. The fuzzy membership function for nutrient satisfaction is defined as

$$\mu_i(X) = \begin{cases} 0, & N_i(X) \leq L_i, \\ \dfrac{N_i(X) - L_i}{U_i - L_i}, & L_i < N_i(X) < U_i, \\ 1, & N_i(X) \geq U_i. \end{cases} \tag{8}$$

where

$$N_i(X) = \sum_{j=1}^{n} a_{ij}\, x_j. \tag{9}$$

For cost satisfaction, the membership function is

$$\mu_C(X) = \begin{cases} 1, & C(X) \leq C_{\min}, \\ \dfrac{C_{\max} - C(X)}{C_{\max} - C_{\min}}, & C_{\min} < C(X) < C_{\max}, \\ 0, & C(X) \geq C_{\max}. \end{cases} \tag{10}$$

where $C(X) = Z_1(X)$.

For dietary quality, the satisfaction function is

$$\mu_Q(X) = \begin{cases} 0, & Q(X) \leq Q_{\min}, \\ \dfrac{Q(X) - Q_{\min}}{Q_{\max} - Q_{\min}}, & Q_{\min} < Q(X) < Q_{\max}, \\ 1, & Q(X) \geq Q_{\max}. \end{cases} \tag{11}$$

where $Q(X) = Z_2(X)$.

For sodium reduction, the satisfaction function is

$$\mu_S(X) = \begin{cases} 1, & S(X) \leq S_{\min}, \\ \dfrac{S_{\max} - S(X)}{S_{\max} - S_{\min}}, & S_{\min} < S(X) < S_{\max}, \\ 0, & S(X) \geq S_{\max}. \end{cases} \tag{12}$$

The overall satisfaction level may be represented as

$$\lambda = \min\{\mu_C(X), \mu_Q(X), \mu_S(X), \mu_1(X), \dots, \mu_m(X)\}. \tag{13}$$

5. Multi-Objective Fuzzy Goal Programming Model

Goal programming is used to transform multiple conflicting objectives into a single optimization model. Let g_k be the target value of objective k, and let d_k^+ and d_k^- be the positive and negative deviation variables, respectively.

The general goal constraint is

$$f_k(X) + d_k^- - d_k^+ = g_k, \quad k = 1,2,\dots,p. \tag{14}$$

The weighted goal programming objective is

$$\min Z = \sum_{k=1}^{p} w_k\,(d_k^+ + d_k^-), \tag{15}$$

where $w_k \geq 0$ is the importance weight assigned to objective k.

For the proposed dietary planning model, the three goal equations are:

$$Z_1(X) + d_1^- - d_1^+ = g_1, \tag{16}$$

$$Z_2(X) + d_2^- - d_2^+ = g_2, \tag{17}$$
$$Z_3(X) + d_3^- - d_3^+ = g_3. \tag{18}$$

Since cost and sodium are minimization objectives, positive deviations are undesirable. Since quality is a maximization objective, negative deviation is undesirable. Therefore, an improved achievement function is

$$\min Z = w_1 d_1^+ + w_2 d_2^- + w_3 d_3^+ + \sum_{i=1}^{m} \rho_i \left(1 - \mu_i(X)\right), \tag{19}$$

where ρ_i is the penalty weight associated with nutrient satisfaction.

The final model is

$$\min \quad w_1 d_1^+ + w_2 d_2^- + w_3 d_3^+ + \sum_{i=1}^{m} \rho_i \left(1 - \mu_i(X)\right), \tag{20}$$

$$\text{s.t.} \quad Z_1(X) + d_1^- - d_1^+ = g_1, \tag{21}$$
$$Z_2(X) + d_2^- - d_2^+ = g_2, \tag{22}$$
$$Z_3(X) + d_3^- - d_3^+ = g_3, \tag{23}$$
$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i, \quad i = 1, \dots, m, \tag{24}$$
$$L_j \leq x_j \leq U_j, \quad j = 1, \dots, n, \tag{25}$$
$$x_j, d_k^+, d_k^- \geq 0. \tag{26}$$

This section establishes basic mathematical properties of the proposed model.

Theorem 1. If the nutritional requirement set is attainable by at least one combination of available food items and all food costs are finite and non-negative, then the proposed fuzzy goal programming model admits at least one feasible solution.

Proof. Assume that there exists a vector X^0 satisfying $AX^0 \geq B$, $L \leq X^0 \leq U$, and $X^0 \geq 0$. Since the deviation variables d_k^+ and d_k^- are non-negative and free to adjust the goal equations, each goal equation can be satisfied by appropriate deviation values. Therefore, X^0 together with suitable deviation variables forms a feasible solution of the fuzzy goal programming model [9]. ▫

Theorem 2. The deterministic feasible region generated by the nutritional and food availability constraints is convex.

Proof. Each nutritional constraint is a linear inequality and each availability constraint is also linear. The feasible region is the intersection of finitely many half-spaces and bounded intervals. Since half-spaces and intervals are convex sets, their intersection is convex. ▫

Theorem 3. The weighted goal programming achievement function is continuous over the feasible dietary region.

Proof. The cost, quality, sodium, and nutrient functions are linear in the decision variables. The deviation terms are linear, and the membership functions are piecewise linear. A finite weighted sum of continuous piecewise linear functions is continuous. Hence, the achievement function is continuous over the feasible dietary region. ▫

Remark 1. Convexity and continuity support computational stability and allow the model to be solved using standard mathematical programming techniques after appropriate linearization of piecewise membership functions.

3. Results

The creation of a representative dataset involved commonly available food items that can be connected with hypertension diet planning. The nutrients studied here are calcium, potassium, magnesium, protein, dietary fiber, vitamin E, and sodium. Table 1 below provides a simplified nutritional database utilized for the numerical experiment [10].

Table 1. Representative nutritional database per serving unit

Food	Cost	Quality	Sodium	Calcium	Potassium	Protein	Fiber
Chicken breast	0.80	7.5	70	12	256	31.0	0.0
Brown rice	0.35	6.2	5	10	86	2.6	1.8
Soybeans	0.45	8.0	2	102	515	16.6	6.0
Whole wheat	0.40	7.0	4	34	405	13.7	12.2
Spinach	0.55	9.0	79	99	558	2.9	2.2
Orange	0.30	7.8	0	43	181	0.9	2.4
Low-fat milk	0.60	8.3	100	120	150	3.4	0.0
Potato	0.25	6.5	6	8	328	1.7	1.8

The objective weights used in the experiment are shown in Table 2. These weights can be modified according to decision-maker preference.

Table 2. Objective weights used in the fuzzy goal programming model

Objective	Symbol	Weight
Cost minimization	w_1	0.40
Quality maximization	w_2	0.35
Sodium minimization	w_3	0.25

Table 3 gives a representative optimized dietary allocation.

Table 3. Optimized dietary allocation

Food item	Quantity (servings/day)	Approximate contribution
Chicken breast	0.80	Protein source
Whole wheat	1.20	Fiber and magnesium source
Spinach	2.50	Potassium and calcium source
Low-fat milk	1.00	Calcium source
Orange	1.00	Potassium and vitamin source
Soybeans	0.60	Plant protein source
Potato	0.40	Potassium support

The solution balances cost, nutrient quality, and sodium reduction. It avoids relying on a single food item and produces a more realistic allocation than a pure cost-minimization model [11].

Sensitivity analysis was conducted to examine the robustness of the model under practical variations. Four scenarios were considered:

1. A uniform 10% increase in food prices.
2. A 5% increase in nutritional requirements.
3. Limited availability of leafy green vegetables.
4. A 20% reduction in the allowable sodium threshold.

Table 4. Sensitivity analysis results

Scenario	Cost index	Quality index	Satisfaction level
Baseline	1.000	0.920	0.910
Price +10%	1.100	0.918	0.902
Requirements +5%	1.145	0.935	0.895
Limited spinach	1.230	0.902	0.870
Reduced sodium	1.180	0.910	0.940

In the case of a price-hike situation, one can see increased costs but made no changes in the general composition of their eating pattern. With more requirements, one needs to consume more food items with essential nutritional benefits. The smallest level of satisfaction is dictated by the fact that due to the shortage of leafy greens; one has less access to foods that have many wholesome nutrients contained in them. Decreased sodium content increases the level of satisfaction in terms of the health aspect, however increases the cost of consumption as a better type of food should be used instead of highly-sodium products [12].

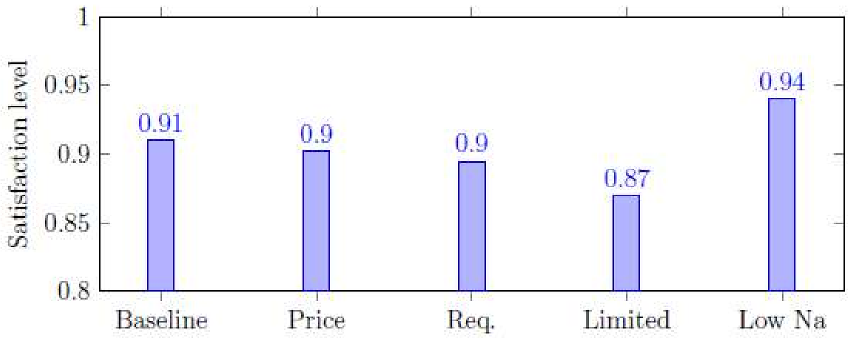

Figure 1. Satisfaction levels under sensitivity scenarios

A comparative evaluation was conducted among three modeling approaches: classical linear programming, deterministic goal programming, and the proposed fuzzy goal programming model.

Table 5. Comparison of optimization approaches

Criterion	LP	Goal Programming	Proposed FGP
Cost minimization	High	Medium	Medium
Multiple objectives	Low	High	High
Uncertainty handling	Low	Low	High
Dietary flexibility	Low	Medium	High
Interpretability	High	High	High
Personalization capacity	Low	Medium	High

Although the deterministic linear programming (LP) model is easy to use and computationally effective, it mainly emphasizes cost and fixed restrictions. The goal programming method improves flexibility by incorporating various goals. The proposed fuzzy goal programming method is highly flexible since it enables the partial fulfillment of uncertain dietary needs as well as multiple competing objectives [13].

4. Discussion

The obtained results demonstrate that fuzzy goal programming can be a useful instrument in the construction of a decision support system for dietary planning for patients suffering from hypertension. This approach allows the constraints to be viewed as divergent and uncertain characteristics, unlike a deterministic diet model. This characteristic is of great importance because clinical prescriptions, the constituent of foods, as well as the needs of the patient can change on time [14].

Another advantage of the suggested model as compared to pure cost minimization is in the fact that least cost diet could be not only fulfilling a series of mathematical constraints but also could be substantially unrealistic, uninteresting, or nutritionally unbalanced. The model develops more sensible solutions due to the presence of quality and sodium target points [15].

It should also be noted that a theoretical analysis allows making a conclusion about the existence of important mathematical properties of the model, namely, convexity and continuity. The convexity of feasible region allows ensuring the stabilization of calculations and consequently making them possible.

The analysis of complexity indicates that the model can respond well to shifts in food prices, tougher health regulations and constraints on food availability. This information is vital in healthcare management in conditions where the markets do not perform consistently while patients may be unable to access certain products. The amount of sodium reduction in the model points out that higher health goals are likely to involve higher expenses [16].

The framework presented here may be further developed including such aspects as calorie count, saturated fats, cholesterol, glycemic index, etc. Using machine learning and mobile health platforms as well can enhance personalization capabilities of the framework further.

5. Conclusion

An innovative multi-objective fuzzy goal programming approach to dietary planning for hypertension has been introduced in this study. This approach includes pushing the boundaries of dietary cost, nutritional value, sodium consumption, and ensuring nutrient satisfaction.

In order to reflect the uncertainty in nutritional requirements and satisfaction, fuzzy sets have been introduced, while the goal programming has provided us with the trade-off concerning opposed objectives.

The theoretical analysis produced the feasibility conditions and proved that the feasible region is convex and the achievement function is continuous. The computational experiments showed that the approach can provide dietary options that would be beneficial in terms of hypertension treatment.

The sensitivity analysis proved that the approach is flexible and effective and can be successfully applied to different conditions with price change, increased nutritional requirements, limited food supply, and sodium restriction.

The research contributes significantly to mathematical healthcare decision support and offers a versatile and interpretable optimization procedure. Despite the fact this procedure cannot replace the clinical judgment, it is able to help nutritionists and researchers in developing affordable dietary solutions. It is very likely that further developments could include stochastic uncertainty analysis, real data, multi-period meal plan generation, and hybridized met.

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