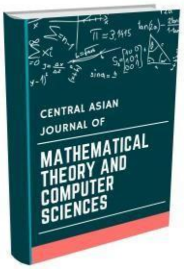

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Article

On Weakly Nearly Primary (W.N.P) Submodules and its Characteristics

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Abstract: Investigating characterizations of weakly Nearly primary submodules in terms of specific module types is the aim of this paper. With the aid of several module types, including, we offer characterizations for the class of multiplication modules. Additionally, we include a few requirements to demonstrate the residual of a weakly Nearly Primary ideal is a Nearly Primary submodule. The characteristics of modules or submodules were also explained, along with their impact on theorems, properties, and fulfillment of conditions.

Keywords: Primary, Nearly Primary, Weakly Nearly Primary.

1. Introduction

Lu first proposed the idea of primary submodule, a generalization of prime submodule some 34 years ago, Al-Ashraf, $ƀ \le Ƒ$, $ƀ$ is named primary if $ɴȩ \in ƀ$, for $ɴ \in \mathcal{R}$, $ȩ \in Ƒ$ then, $ȩ \in ƀ$ or $ɴ \in \sqrt{[ƀ:_{\mathcal{R}} Ƒ]}$[1, 2]. If $ƀ \le Ƒ$ is named weakly primary if $0 \neq ɴȩ \in ƀ$, for $ɴ \in \mathcal{R}$, $ȩ \in Ƒ$ then, $ȩ \in ƀ$ or $ɴ \in \sqrt{[ƀ:_{\mathcal{R}} Ƒ]}$ [3, 4, 5]. Hashim and others generalized this concept to Nearly primary in shape if $ɴȩ \in ƀ$, for $ɴ \in \mathcal{R}$, $ȩ \in Ƒ$, then $ȩ \in ƀ + J(Ƒ)$ or $ɴ \in \sqrt{[ƀ + J(Ƒ):_{\mathcal{R}} Ƒ]}$[6, 7, 8]. Ajeel, Abdul-AlKalik and others They had recently studied concepts of interest and relevance [9, 10, 11] . Athab explained the new concept J-primary submodule [12]. In addition, there is a wealth of research that positively relates to clarifying related concepts such as. Ahmed, Al-Ragab, Dahash [13, 14, 15]. We present characterizations for numerous modules in this paper's primary results section, we show, under certain conditions, the residual of weakly Nearly primary submodule is a weakly Nearly primary ideal. In addition, we have explained some of the characteristics and conditions related to this topic

2. Research Methodology

This paper analyzes Weakly Nearly Primary (W.N.P.) submodules using a rigorous deductive method in abstract algebra. It produces innovative discoveries through systematic mathematical proofs and provides exact definitions based on well-established module theory. Standard methods, such as contradiction and direct demonstration, are methodically used, and specific instances are given to support and elucidate the theoretical conclusions.

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3. Results and discussion

A set of conclusions, characteristics, and examples that provide sufficient clarification for the above concept.

Proposition (1):

$ƀ \leq Ƒ$. ƀ is weakly Nearly primary (W.N.P) submodule $\Leftrightarrow [ƀ:_{Ƒ} ɴ] \subseteq [ƀ+J(Ƒ):_{Ƒ} ɴ^n] \cup [0:_{Ƒ} ɴ]$ for $ɴ \in \mathcal{R}, n \in Z^+$.

Proof

First direction: if $ẹ \in [ƀ:_{Ƒ} ɴ]$, $0 \neq ɴẹ \in ƀ$, $ẹ \in Ƒ$. We assume $ẹ \notin ƀ + J(Ƒ)$. We conclude from $ẹ \in [ƀ:_{Ƒ} ɴ]$, $0 \neq ɴẹ \in ƀ \Longrightarrow ẹ \in [ƀ:_{Ƒ} ɴ] \subseteq [ƀ+J(Ƒ):_{Ƒ} ɴ^n] \cup [0:_{Ƒ} ɴ] \Longrightarrow ẹ \in [ƀ+J(Ƒ):_{Ƒ} ɴ^n] \Longrightarrow ẹɴ^n \in [ƀ + J(Ƒ)] \Longrightarrow ɴ^n Ƒ \subseteq [ƀ + J(Ƒ)] \Longrightarrow ɴ^n \in [ƀ+J(Ƒ):_R Ƒ]$ and $ẹ \in \sqrt{[ƀ+J(Ƒ):_R Ƒ]}$. Therefore ƀ is (W.N.P) of Ƒ.

second direction: if $ẹ \in [ƀ:_{Ƒ} ɴ]$ and $ẹ \notin (ƀ + J(Ƒ)) \Longrightarrow ɴẹ \in ƀ$.

first case: If $ɴẹ = 0 \Longrightarrow ẹ \in [0:_{Ƒ} ɴ]$ then $ẹ \in [ƀ+J(Ƒ):_{Ƒ} ɴ^n] \cup [0:_{Ƒ} ɴ]$.

second case: If $ɴẹ \neq 0 \in ƀ$, ƀ is (W.N.P) also $ẹ \notin ƀ + J(Ƒ) \Longrightarrow ɴ^n \in [ƀ+J(Ƒ):_R Ƒ] \Longrightarrow. ɴ^n Ƒ \subseteq [ƀ + J(Ƒ)]$, Who is this $ẹɴ^n \in [ƀ + J(Ƒ)] \Longrightarrow ẹ \in [ƀ+J(Ƒ):_{Ƒ} ɴ^n]$ certainly $ẹ \in [ƀ:_{Ƒ} ɴ] \subseteq [ƀ+J(Ƒ):_{Ƒ} ɴ^n] \cup [0:_{Ƒ} ɴ] \Longrightarrow [ƀ:_{Ƒ} ɴ] \subseteq [ƀ+J(Ƒ):_{Ƒ} ɴ^n] \cup [0:_{Ƒ} ɴ]$.

Proposition (2):

$ƀ \leq Ƒ$ and $[ƀ+J(Ƒ):_{\mathcal{R}} Ƒ]$ is a maximal semi-prime ideal of $\mathcal{R}$. Then ƀ is (W.N.P) submodule of Ƒ.

Lemma (3): [1]

$\mathcal{B}$ ideal of a ring $\mathcal{R}$. Then $\mathcal{B}$ is a maximal iff $\mathcal{B} + < h > = \mathcal{R}$, $\forall\, h \notin \mathcal{B}$.

Lemma (4): [1]

An ideal $\mathcal{B}$ is a semi.prime. iff $\sqrt{\mathcal{B}} = \mathcal{B}$.

Proof

From (lemma 4) $[ƀ+J(Ƒ):_{\mathcal{R}} Ƒ] = \sqrt{[ƀ+J(Ƒ):_{\mathcal{R}} Ƒ]}$ because $[ƀ+J(Ƒ):_R Ƒ]$ is a maximal semi-prime. If $ɴẹ \neq 0 \in ƀ$ for $ɴ \in \mathcal{R}$, $ẹ \in Ƒ$ with $ɴ \notin \sqrt{[ƀ+J(Ƒ):_{\mathcal{R}} Ƒ]} = [ƀ+J(Ƒ):_R Ƒ]$. From the information provided $[ƀ+J(Ƒ):_{\mathcal{R}} Ƒ]$ is maximal $\Longrightarrow \sqrt{[ƀ+J(Ƒ):_{\mathcal{R}} Ƒ]}$ is maximal. By lemma (3) $\mathcal{R} = <ɴ> + \sqrt{[ƀ+J(Ƒ):_R Ƒ]}$, for $< ɴ >$ is an ideal of R $\Longrightarrow \mathcal{R} = < ɴ > + [ƀ+J(Ƒ):_{\mathcal{R}} Ƒ]$, that means $1 = h\,ɴ + k$, $\exists h \in \mathcal{R}$, $k \in [ƀ+J(Ƒ):_{\mathcal{R}} Ƒ]$. Therefore $ẹ = hɴẹ + kẹ \in [ƀ + J(Ƒ)]$. Then ƀ is (W.N.P) submodule of Ƒ.

Proposition (5):

Suppose that $J(Ƒ)$ is a weakly primary submodule of Ƒ. If $ƀ \leq Ƒ$ with $ƀ \ll Ƒ$. Then ƀ is (W.N.P) submodule of Ƒ.

Proof

Let $ɴẹ \neq 0 \in ƀ$, where $ɴ \in \mathcal{R}$, $ẹ \in Ƒ$. From imposition $ɴẹ \neq 0 \in ƀ + J(Ƒ)$, by $ƀ \ll Ƒ \Longrightarrow ƀ \subseteq J(Ƒ)$ that is $ɴẹ \neq 0 \in J(Ƒ)$. Since $J(Ƒ)$ is a weakly primary of Ƒ, then $ẹ \in J(Ƒ) \subseteq ƀ + J(Ƒ)$ or $ɴ^n Ƒ \subseteq J(Ƒ) \subseteq ƀ + J(Ƒ)$. That is $ẹ \in ƀ + J(Ƒ)$ or $ɴ \in \sqrt{[ƀ+J(Ƒ):_{\mathcal{R}} Ƒ]}$. ƀ is (W.N.P) submodule of Ƒ.

Proposition (6):

If H, $K \leq Ƒ$ with H proper subset of K. If K is (W.N.P) submodule of Ƒ, then $\frac{K}{H}$ is (W.N.P) submodule of $\frac{Ƒ}{H}$.

Proof

If K is (W.N.P) submodule of Ƒ and $0 \neq ɴ\,(ẹ + H) = ɴẹ + H \in \frac{K}{H}$ where $ẹ + H \in \frac{Ƒ}{H}$, $ẹ \in Ƒ$, $ɴ \in \mathcal{R}$, indicates that $ɴẹ \in K$.

If $ɴẹ = 0 \Longrightarrow ɴ\,(ẹ + H) = 0 \Longrightarrow \perp$, so $0 \neq ɴẹ \in K$. Now K is (W.N.P) consequently, $ẹ \in K + J(Ƒ)$ or $ɴ^n Ƒ \subseteq K + J(Ƒ)$. Thus $ẹ + H \in \frac{K + J(Ƒ)}{H}$ or $ɴ^n Ƒ H \subseteq \frac{K + J(Ƒ)}{H}$, consequently, $ẹ + H \in \frac{K}{H} + \frac{K + J(Ƒ)}{H} \subseteq \frac{K}{H} + J\left(\frac{Ƒ}{H}\right)$ or $ɴ^n \frac{Ƒ}{H} \subseteq \frac{K}{H} + \frac{K + J(Ƒ)}{H} \subseteq \frac{K}{H} + J\left(\frac{Ƒ}{H}\right)$. That is $ẹ + H \in \frac{K}{H} + J\left(\frac{Ƒ}{H}\right)$ or $ɴ \in \sqrt{\left[\frac{K}{H} + J\left(\frac{Ƒ}{H}\right) :_{\mathcal{R}} \frac{Ƒ}{H}\right]}$. Thus $\frac{K}{H}$ is (W.N.P) submodule of $\frac{Ƒ}{H}$.

Proposition (7):[24]

$Ƒ$ be a semi. simple. then $J(Ƒ) = 0$

Proposition (8):

Let $Ƒ$ be a semi. simple. module and $ƅ$ is (W.N.P) submodule of $Ƒ$. Then $[ƅ :_{\mathcal{R}} Ƒ]$ is a (W.N.P) ideal of $\mathcal{R}$.

Proof

Assume that $ɴẹ \neq 0 \in [ƅ :_{\mathcal{R}} Ƒ]$, for $ɴ, ẹ \in \mathcal{R} \Longrightarrow 0 \neq ɴ(ẹƑ) \subseteq ƅ$. $ƅ$ is (W.N.P) submodule of $Ƒ$, $ẹƑ \subseteq ƅ + J(Ƒ)$ or $ɴ^n Ƒ \subseteq ƅ + J(Ƒ)$. Now $Ƒ$ be a semi. simple. then $J(Ƒ) \subseteq ƅ$, then $ƅ = ƅ + J(Ƒ)$, consequently $ẹƑ \subseteq ƅ$ or $ɴ^n Ƒ \subseteq ƅ$, that is $ẹ \in [ƅ :_{\mathcal{R}} Ƒ] \subseteq [ƅ :_{\mathcal{R}} Ƒ] + J(Ƒ)$ or $ɴ^n \in [ƅ :_{\mathcal{R}} Ƒ] \subseteq [ƅ :_{\mathcal{R}} Ƒ] + J(Ƒ) = [[ƅ :_{\mathcal{R}} Ƒ] + J(Ƒ) :_{\mathcal{R}} \mathcal{R}]$. So $ẹ \in [ƅ :_{\mathcal{R}} Ƒ] + J(Ƒ)$ or $ɴ \in \sqrt{[[ƅ + J(Ƒ) :_{\mathcal{R}} \mathcal{R}]}$. Hence $[ƅ :_{\mathcal{R}} Ƒ]$ is a (W.N.P) ideal of $\mathcal{R}$.

The following example illustrates why the opposite of the aforementioned statement is not generally true.

Example (9):

The Ƶ-module $Ƶ \oplus Ƶ$ contain the submodule $ƅ = 4Ƶ \oplus (0)$, then $[4Ƶ \oplus (0) :_{Ƶ} Ƶ \oplus Ƶ] = (0)$ is (W.N.P) ideal of Ƶ. It is known that (0) is (W.N.P) ideal of Ƶ. Now if $4 \in Ƶ$ and $(3,0) \in Ƶ \oplus Ƶ$, obviously $ƅ = 4Ƶ \oplus (0)$ is not (W.N.P) $Ƶ \oplus Ƶ$, because $(0,0) \neq 4(3,0) = (0,0) \in 4Ƶ \oplus (0)$, but neither $(3,0) \in (4Ƶ \oplus (0) + J(Ƶ))$ nor $4 \in \sqrt{[(4Ƶ \oplus (0) + J(Ƶ)) :_{Ƶ} Ƶ \oplus Ƶ]} = \sqrt{[(4Ƶ \oplus (0) + 0) :_{Ƶ} Ƶ \oplus Ƶ]} = \sqrt{< 0 >} =$<0>.

Remark (10):

The above property is realized in two directions if the module $Ƒ$

- Ƶ.Regular $\mathcal{R}$.module.
- Faithful multiplication $\mathcal{R}$.module.
- Non.singular multiplication $\mathcal{R}$.module.
- Projective multiplication $\mathcal{R}$.module.

Proposition (11):

$ƅ \leq Ƒ$ with $Ƒ$ be a semisimple module and $ƅ$ is (W.N.P) submodule of $Ƒ$, then $ƅ[\varkappa]$ (the collection of all polynomials with coefficients in ƅ) is (W.N.P) submodule of $Ƒ[\varkappa]$.

Proof

Suppose that $\mu : Ƒ[\varkappa] \longrightarrow (\frac{Ƒ}{ƅ})[\varkappa]$ is an $\mathcal{R}$-epimorphism described by $\mu(ɷ_1\varkappa_1 + ɷ_2\varkappa_2 + \ldots + ɷ_\eta\varkappa_\eta) = (ɷ_1 + ƅ)\varkappa + (ɷ_2 + ƅ)\varkappa^2 + \ldots + (ɷ_\eta + ƅ)\varkappa^\eta$, where $ɷ_1, ɷ_2, \ldots, ɷ_\eta \in Ƒ$ and $ɷ_1 + ƅ, ɷ_2 + ƅ, \ldots, ɷ_\eta + ƅ \in \frac{Ƒ}{ƅ}$. The kernel of μ is acquired by lowering the coefficient module $ƅ$, obviously $\frac{Ƒ[\varkappa]}{ƅ[\varkappa]} \cong (\frac{Ƒ}{ƅ})[\varkappa]$. Since $(0) \neq \frac{Ƒ}{ƅ} = ƅ$, implies that $\frac{Ƒ[\varkappa]}{ƅ[\varkappa]} \neq (0) = ƅ[\varkappa]$. If $\exists$ polynomial $(ɷ_1 + ƅ)\varkappa + (ɷ_2 + ƅ)\varkappa^2 + \ldots + (ɷ_\eta + ƅ)\varkappa^\eta$ in $(\frac{Ƒ}{ƅ})[\varkappa]$ and $ʋ$ be a zero divisor of $(\frac{Ƒ}{ƅ})[\varkappa]$, such that $ʋ((ɷ_1 + ƅ)\varkappa + (ɷ_2 + ƅ)\varkappa^2 + \ldots + (ɷ_\eta + ƅ)\varkappa^\eta) = (0)$, thus $\exists\, 1 \leq j \leq n$ with $ʋ(ɷ_j + ƅ) = 0 = ƅ$ and $ɷ_j + ƅ \neq 0 = ƅ$, it follows that $ʋɷ_j \in ƅ$ and $ɷ_j \notin ƅ + J(Ƒ)$.

If $ʋɷ_j = 0$ then $ʋ \in [0 :_{\mathcal{R}} ɷ_j]$.

If $0 \neq ʋɷ_j \in ƅ$ with $ɷ_j \notin ƅ + J(Ƒ)$ and ƅ is (W.N.P) submodule of Ƒ, implies that $ʋ \in\sqrt{[ƅ+J(Ƒ):_{\mathcal{R}} Ƒ]}$, that is $ʋ^n Ƒ \subseteq ƅ + J(Ƒ) = ƅ$ (Ƒ be a semisimple), thus $ʋ^n Ƒ \subseteq ƅ$, so $ʋ^n \frac{Ƒ}{ƅ} = (0)$, therefore $ʋ\in\sqrt{\left[0:_{\mathcal{R}} \frac{Ƒ}{ƅ}\right]}$. Hence $ʋ \in [0:_{\mathcal{R}} ɷ_j] \cup \sqrt{\left[0:_{\mathcal{R}} \frac{Ƒ}{ƅ}\right]}$.

Obviously $(\frac{Ƒ}{ƅ}) \subseteq (\frac{Ƒ}{ƅ})[ϰ]$. Thus $ƅ[ϰ]$ is (W.N.P) submodule of $Ƒ[ϰ]$.

Proposition (12):

Let Ƒ be a secondary $\mathcal{R}$-module, $J(Ƒ) \subseteq ƅ$ with $0 \neq ƅ$ is (W.N.P) submodule of Ƒ. Then ƅ is a secondary.

Proof

Let $ʋ^n Ƒ = (0), ʋ \in \mathcal{R},\ n \in Ƶ^+$, then $ʋ^n ƅ \subseteq ʋ^n Ƒ = (0)$, implies that $ʋ^n ƅ = 0$. Assume that $ʋƑ = Ƒ$, we demonstrate that $ʋƅ = ƅ$. Let $0 \neq ɴ$ element in ƅ, it follows that $ɴ = ʋɇ$ for some $ɇ \in Ƒ$. That is $0 \neq ʋɇ \in ƅ$ and $ʋ^n Ƒ \nsubseteq ƅ + J(Ƒ) = ƅ$, that is $ʋ^n Ƒ \nsubseteq ƅ$, Consequently $ɇ \in ƅ + J(Ƒ) = ƅ$, Consequently, $ɇ\in ƅ$. So, $ɴ = ʋɇ \in ʋƅ$. Thus $ʋƅ = ƅ$. Hence ƅ is a secondary.

Proposition (13):

If $ƅ \nsubseteq ⱳ$ with ƅ, ⱳ are (W.N.P) submodules of Ƒ and $J(Ƒ) \subseteq ƅ$ or $J(Ƒ) \subseteq ⱳ$. Then $(ƅ \cap ⱳ)$ is (W.N.P) submodules of Ƒ.

Proof

It sure is $(ƅ \cap ⱳ) \leq Ƒ$. Let $0 \neq ɴɇ \in (ƅ \cap ⱳ)$, where $ɴ \in \mathcal{R}$, $ɇ \in Ƒ$, with $ɴ^n \notin [(ƅ \cap ⱳ) + J(Ƒ) :_{\mathcal{R}} Ƒ]$, $n \in Ƶ^+$, that is $ɴ^n Ƒ \nsubseteq (ƅ \cap ⱳ) + J(Ƒ)$, Consequently $ɴ^n Ƒ \nsubseteq ƅ + J(Ƒ)$ and $ɴ^n Ƒ \nsubseteq ⱳ + J(Ƒ)$. So, $ɴɇ \neq 0 \in H$ and $ɴɇ \neq 0 \in D$. Since ƅ, ⱳ are (W.N.P) submodules of Ƒ, it follows that

$ɴ\in ƅ + J(Ƒ)$ and $ɴ\in ⱳ + J(Ƒ)$, it follows that $ɴ \in (ƅ + J(Ƒ)) \cap (ⱳ + J(Ƒ))$. Now, $J(Ƒ) \subseteq ⱳ$, then $ⱳ + J(Ƒ) = ⱳ$. Thus $ɴ \in(ƅ + J(Ƒ)) \cap ⱳ$, by modular law $ɴ \in(ƅ \cap ⱳ) + J(Ƒ)$. Therefore $(ƅ \cap ⱳ)$ is (W.N.P) submodules of Ƒ.

Remark (14):

Let $ƅ \leq Ƒ\ and\ ⱳ \leq Ƒ$ with $ƅ \cong ⱳ$. If ƅ is (W.N.P) submodules of Ƒ, It is not essential ⱳ is (W.N.P) submodules of Ƒ.

The example that follows demonstrates that:

Consider the Ƶ-module Ƶ and $ƅ = \langle 2\rangle$, $ⱳ = \langle 6\rangle$, we have $ƅ = \langle 2\rangle$ is (W.N.P) of Ƶ-module Ƶ and $ƅ= \langle 2\rangle \cong ⱳ=\langle 6\rangle$, but $\langle 6\rangle$ is not (W.N.P), To clarify $0 \neq 2.3 \in \langle 6\rangle$, where $2,3 \in Ƶ$, but neither $3 \in \langle 6\rangle+ J(Ƶ)=\langle 6\rangle+\langle 0\rangle=\langle 6\rangle$, nor $3 \in \sqrt{[\langle 6\rangle + J(Ƶ)) :_Ƶ Ƶ]} =\sqrt{[\langle 6\rangle + \langle 0\rangle) :_Ƶ Ƶ]} =\sqrt{[\langle 6\rangle:_Ƶ Ƶ]} = 6Ƶ$.

4. Conclusions

This work examined weakly nearly primary (W.N.P) submodules and established several characterizations that explain their algebraic structure and behavior within different module types. The results showed that W.N.P submodules form an important generalization connecting primary and nearly primary concepts through conditions involving annihilators and the Jacobson radical.

It was demonstrated that the W.N.P property is preserved under quotient modules, residual ideals, and polynomial extensions when suitable structural assumptions are satisfied, particularly in semisimple and multiplication modules. The study also clarified the essential role of the Jacobson radical in determining when weakly nearly primary submodules lead to stronger algebraic properties. Overall, the findings provide a clearer theoretical framework for understanding generalized primeness in module theory and offer a basis for further investigations into related module structures.

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