



Article

# Some Properties of Fuzzy Ideals of Semi-Weakly $m$ -Regular LA Semi Group

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**Abstract:** In this work we present the Some properties fuzzy ideals for the semi-weakly  $m$  regular for a LA semi-group as well as some important results are studying such as If  $\varpi$  is a fuzzy-interior ideal in Semi-weakly  $m$  regular LAsemi-g  $\mathcal{F}$ , with identity, then  $\varpi(\alpha^m \uparrow^m) = \varpi(\uparrow^m \alpha^m)$  for all  $\alpha, \uparrow \in \mathcal{F}$ ,  $m$  is a constant and A fuzzy-subset  $\varpi$  of Semi-weakly  $m$  regular LAsemi-g  $\mathcal{F}$  with left identity be the fuzzy-left ideals in  $\mathcal{F}$  iff it is the fuzzy-right ideals for the  $\mathcal{F}$ . Many properties and examples for a fuzzy-ideals(right, left, two sided, quasi, interior, bi-generalized-bi, and (1, 2)) of Semi-weakly  $m$  regular of LA semi-group were shown.

**Keywords:** LA semi-group, semi-weakly  $m$  regular, fuzzy-two sided ideal, fuzzy- interior ideal.

## 1. Introduction

In (2012) Zubayda M I, Maha F K interduced the concepted  $m$  regular semi-group  $\mathcal{F}$  which is  $u \in \mathcal{F}$ , there exists  $v \in \mathcal{F}$ , s.t  $u^m = u^m v u^m$ , where  $m$  is constant[1].

The concept of  $S$  weakly regular semi-groups was first introduced by Akram, S. M, Which is  $u \in \mathcal{F}$ , there exists  $v, w \in \mathcal{F}$ , s.t  $u = u v u^2 w$  [2].

In this paper we have extended the concept of Semi-weakly  $m$  regular LAsemi-group  $\mathcal{F}$  which is  $u \in \mathcal{F}$ , there exists  $v, w \in \mathcal{F}$ , s.t  $u^m = u^m v u^{2m} w$ , where  $m$  is constant]. They named it as a left almost semi-group (LAsemi-g).

An LAsemi-g be the groupoid  $\mathcal{F}$  whose elements a left invertive laws  $(uv)w = (wv)u$  for all  $u, v, w \in \mathcal{F}$  [3].

In an LAsemi-g the medial's laws  $(uv)(wz) = (uw)(vz)$  for all  $u, v, w, z \in \mathcal{F}$  [3].

An LAsemi-g may or may not includes the left identities.

An LAsemi-g with left identity the paramedical laws  $(uv)(wz) = (zw)(vu)$  hold, [3]. If an LAsemi-g includes the left identity, so by use medial laws, implies that  $u(vw) = v(uw)$  [3].

The fuzzy-subsets  $\varpi$  in the semi-g  $\mathcal{F}$  be named fuzzy-sub semi-g for the  $\mathcal{F}$  if  $\varpi(uv) \geq \min \{\varpi(u), \varpi(v)\}$ [4].

The fuzzy sub semi-g  $\varpi$  in the semi-g  $\mathcal{F}$  named fuzzy-interior ideals for the  $\mathcal{F}$  where  $\varpi(uvw) \geq \varpi(v)$  [5].

If  $\varpi$  fuzzy-left ideal, fuzzy-right ideals for the  $\mathcal{F}$ , so  $\varpi$  be the fuzzy-ideals for the  $\mathcal{F}$

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[6].

If  $\mathcal{F}$  is the order semi-g, the fuzzy-subsemi-g  $\varpi$  for the  $\mathcal{F}$  is fuzzy-left (resp, right)ideals for the  $\mathcal{F}$  if 1-  $\varrho \leq \alpha \Rightarrow \varpi(\varrho) \geq \varpi(\alpha)$  2-  $\varpi(\varrho\alpha) \geq \varpi(\alpha)$ ,  $(\varpi(\varrho\alpha) \geq \varpi(\varrho))$  for all  $\varrho, \alpha \in \mathcal{F}$  [6].

Let  $\mathcal{F}$  is ordered semi-g, the fuzzy-subsemi-g  $\varpi$  for the  $\mathcal{F}$  be the fuzzy-bi ideals for the  $\mathcal{F}$  where 1-  $\varrho \leq \alpha \Rightarrow \varpi(\varrho) \geq \varpi(\alpha)$  2-  $\varpi(\varrho\alpha w) \geq \min\{\varpi(\varrho), \varpi(w)\}$  for all  $\varrho, \alpha, w \in \mathcal{F}$  [7].

If  $\varpi$  is both a fuzzy left-ideal & right ideals for the  $\mathcal{F}$  so  $\varpi$  be the fuzzy two sided ideals for the  $\mathcal{F}$  [7].

The fuzzy-subsemi-g  $\varpi$  for the LAsemi-g  $\mathcal{F}$  is said to be fuzzy-(1, 2) ideal of  $\mathcal{F}$  if  $\varpi((\varrho)(\alpha w)) \geq \varpi(\varrho) \wedge \varpi(\alpha) \wedge \varpi(w)$  for all  $\varrho, \alpha, w \in \mathcal{F}$  [7].

The fuzzy subsets  $\varpi$  for the LAsemi-g  $\mathcal{F}$  is named fuzzy-quasi ideals for the  $\mathcal{F}$  where  $(\varpi \circ K_{\mathcal{F}}) \cap (K_{\mathcal{F}} \circ \varpi) \subseteq \varpi$  [7].

For an ordered semi-g  $\mathcal{F}$  the fuzzy-subsets 0 & 1 in the  $\mathcal{F}$  define

$$0 : \mathcal{F} \rightarrow [0, 1] \mid x \rightarrow 0(x) = 0,$$

$$1 : \mathcal{F} \rightarrow [0, 1] \mid x \rightarrow 1(x) = 1,$$

so it is clear a fuzzy-subset 0 of  $\mathcal{F}$  be least elements for the sets  $(K(\mathcal{F}), \leq)$  that is  $\varpi \circ 0 = 0 \circ \varpi = 0$  [8].

the product  $\varpi \circ \rho$  is defined by:

$$(\varpi \circ \rho)_{(u^m)} = \begin{cases} \bigvee_{u^m = u^m v u^{2m} w} \{\varpi(u^m v) \wedge \rho(u^{2m} w)\}; \text{ for some } v, w \in \mathcal{F}, \text{ s.t } u^m = u^m v u^{2m} w & [9]. \\ 0; & \text{otherwise.} \end{cases}$$

**2. Materials and Methods**

**Definition (1. 1)**

A semi-g  $\mathcal{F}$  is called Semi-weakly  $m$  regular if for all  $\tau \in \mathcal{F}$  there exists  $v, w \in \mathcal{F}$  s.t  $\tau^m = \tau^m v \tau^{2m} w$ , where  $m$  is constant.

**Example (1. 2)**

Let  $\mathcal{F} = \{\tau, v, \beta, s, t\}$  be LAsemi-g with left identity  $s$  with multiplication table.

.	$\tau$	$v$	$\beta$	$s$	$t$
$\tau$	$\tau$	$\tau$	$\tau$	$\tau$	$\tau$
$v$	$\tau$	$v$	$v$	$v$	$v$
$\beta$	$\tau$	$v$	$s$	$t$	$\beta$
$s$	$\tau$	$v$	$\beta$	$s$	$t$
$t$	$\tau$	$v$	$t$	$\beta$	$s$

$\mathcal{F}$  is Semi-weakly  $m$ -regular because  $\tau^3 = \tau^3 v \tau^6 \beta$ ,  
 $v^3 = v^3 \beta v^6 s$ ,  
 $\beta^3 = \beta^3 t v^6 s$ ,  
 $s^3 = s^3 \beta s^6 t$ ,  
 $t^3 = t^3 s t^6 \beta$ , where  $m=3$ .

**Example (1. 3)**

Let  $\mathcal{F} = \{\tau, v, w, s, t\}$  be LAsemi-g with left identity  $s$  with multiplication table.

.	$\tau$	$v$	$w$	$s$	$t$
$\tau$	$\tau$	$u$	$\tau$	$\tau$	$\tau$
$v$	$\tau$	$t$	$t$	$w$	$t$
$w$	$\tau$	$t$	$t$	$v$	$t$
$s$	$\tau$	$v$	$w$	$s$	$t$
$t$	$\tau$	$t$	$t$	$t$	$t$

Note that  $\mathcal{F}$  is not Semi-weakly  $m$  regular for  $v \in \mathcal{F}$ , there does not exist  $x, y \in \mathcal{F}$ , s.t.  $v^m = v^m x v^{2m} y$ .

**Remark (1. 4)**

If  $\varpi$  is a fuzzy subset of a ALsemi-g  $\mathcal{F}$  with left identity then  $\mathcal{F}$  is Semi-weakly  $m$  regular if  $\varpi(u^m) = \varpi(u^{2m})$  holds for all  $u \in \mathcal{F}$ , while the opposite is not true.

**Example (1. 5)**

Let  $\mathcal{F} = \{g, \varphi, w, s, t\}$  be an LAsemi-g with left identity  $s$  with multiplication table.

.	$g$	$\varphi$	$w$	$s$	$t$
$g$	$g$	$g$	$g$	$g$	$g$
$\varphi$	$g$	$\varphi$	$\varphi$	$\varphi$	$\varphi$
$w$	$g$	$\varphi$	$w$	$s$	$t$
$s$	$g$	$\varphi$	$t$	$w$	$s$
$t$	$g$	$\varphi$	$s$	$t$	$w$

Let us consider a Semi-weakly  $m$  regular LAsemi-g  $\mathcal{F}$  in example defined the fuzzy-subsets  $\varpi$  for the  $\mathcal{F}$  by:

$$\varpi(g) = 0.6, \quad \varpi(\varphi) = 0.2 \text{ and}$$

$$\varpi(w) = \varpi(s) = \varpi(t) = 0.9.$$

then it is easy to see that

$$\varpi(g^m) \neq \varpi(g^{2m}) \text{ for } g \in \mathcal{F}, \text{ where } m \text{ is constant.}$$

**Proposition (1. 6)**

If  $\varpi$  is a fuzzy-interior ideal in Semi-weakly  $m$  regular LAsemi-g  $\mathcal{F}$ , with identity, then  $\varpi(\alpha^m \varpi^m) = \varpi(\varpi^m \alpha^m)$  for all  $\alpha, \varpi \in \mathcal{F}$ ,  $m$  is a constant.

**Proof**

$\Rightarrow$   $\varpi$  be the fuzzy-interior ideals for the Semi-weakly  $m$  regular LAsemi-g  $\mathcal{F}$ , if  $\alpha \in \mathcal{F}$  so  $\exists d, \tau \in \mathcal{F}$ , s.t

$$\alpha^m = \alpha^m d \alpha^{2m} \tau, \text{ where } m \text{ is constant.}$$

$$\varpi(\alpha^m) = \varpi(\alpha^m d \alpha^{2m} \tau)$$

$$\begin{aligned}
&= \varpi((\alpha^m d) \alpha^{2m} \tau) \\
&= \varpi(s \alpha^{2m} \tau), \text{ where } s = (\alpha^m d) \\
&\geq \varpi(\alpha^{2m}) \text{ [because } \varpi \text{ is a fuzzy-interior ideal]} \\
\varpi(\alpha^{2m}) &= \varpi(\alpha^m \alpha^m) \\
&= \varpi(\alpha^m \alpha^m d \alpha^{2m} \tau) \\
&= \varpi((\alpha^{2m} d) \alpha^m (\alpha^m \tau)) \\
&= \varpi((s) \alpha^m (t)) \\
\text{where } s &= (\alpha^{2m} d) \text{ and } t = (\alpha^m \tau) \\
&\geq \varpi(\alpha^m) \text{ [because } \varpi \text{ is a fuzzy-interior ideal].} \\
\text{Therefore } \varpi(\alpha^m) &= \varpi(\alpha^{2m}) \\
\varpi(\alpha^m \alpha^m) &= \varpi((\alpha^m \alpha^m)^2) \\
&= \varpi((\alpha^m \alpha^m) (\alpha^m \alpha^m)) \text{ [by the paramedial law [3]]} \\
&= \varpi((\alpha^m \alpha^m) (\alpha^m \alpha^m)) = \varpi(e(\alpha^m \alpha^m) (\alpha^m \alpha^m)) \\
&\geq \varpi(\alpha^m) \text{ [because } \varpi \text{ is a fuzzy-interior ideal]} \\
\text{Hence } \varpi(\alpha^m \alpha^m) &\geq \varpi(\alpha^m) \dots 1 \\
\varpi(\alpha^m \alpha^m) &= \varpi(\alpha^m \alpha^m d \alpha^{2m} \tau) \\
&= \varpi(\alpha^m [\alpha^m d \alpha^{2m} \tau]) \text{ [by using medial law [3]]} \\
&= \varpi(\alpha^m (\alpha^m d \alpha^{2m} \tau)) \\
&= \varpi(\alpha^m \alpha^m d \alpha^{2m} \tau) \\
&= \varpi(e(\alpha^m \alpha^m) (d \alpha^{2m} \tau)) \\
&\geq \varpi(\alpha^m \alpha^m) \\
\varpi(\alpha^m \alpha^m) &\geq \varpi(\alpha^m \alpha^m) \dots 2, \\
\text{hence from 1 and 2 we get} \\
\varpi(\alpha^m \alpha^m) &= \varpi(\alpha^m \alpha^m) \text{ [11].}
\end{aligned}$$

**Proposition (1.7)**

For a fuzzy-subset  $\varpi$  of Semi-weakly  $m$  regular LAsemi-g  $\mathcal{F}$  and  $K$  is a subset of  $\mathcal{F}$ , then  $(K_{\mathcal{F}} \circ \varpi) = \varpi$ .

**Proof**

Since  $\mathcal{F}$  is Semi-weakly  $m$  regular LAsemi-g therefore, for all  $g \in \mathcal{F} \exists \alpha, f \in \mathcal{F}$ , s.t.  $g^m = g^m \alpha g^{2m} f$ , where  $m$  is constant.

$$\begin{aligned}
g^m &= g^m \alpha g^{2m} f \\
&= (g^m (\alpha g^{2m})) f \text{ [by the left invertive law [3]]} \\
&= (f (\alpha g^{2m})) g^m \\
(K_{\mathcal{F}}(\alpha) \circ \varpi)_{(g^m)} &= \bigvee_{g^m = (f(\alpha g^{2m})) g^m} \{K_{\mathcal{F}}(\alpha) ((f(\alpha g^{2m})) g^m) \wedge \varpi(g^m)\} \\
&= \bigvee_{g^m = (f(\alpha g^{2m})) g^m} \{1 \wedge \varpi(g^m)\} = \varpi(g^m).
\end{aligned}$$

Hence  $(K_{\mathcal{F}} \circ \varpi) = \varpi$

**Proposition(1.8)**

In Semi-weakly  $m$  regular semi-g  $\mathcal{F}$ ,  $(K_{\mathcal{F}}(\alpha) \circ \varpi) = \varpi$  and  $(\varpi \circ K_{\mathcal{F}}(\alpha)) = \varpi$  holds for every fuzzy-two sided ideal  $\varpi$  of  $\mathcal{F}$ .

**Proof**

Since  $\mathcal{F}$  is Semi-weakly  $m$  regular LAsemi-g therefore for all  $g \in \mathcal{F} \exists \alpha, f \in \mathcal{F}$ , s.t.  $g^m = g^m \alpha g^{2m} f$ , where  $m$  is constant.

$$\begin{aligned}
g^m &= g^m \alpha g^{2m} f \\
&= (g^m (\alpha g^{2m})) f \text{ [by the left invertive law [3]]} \\
&= (f (\alpha g^{2m})) g^m \\
&= ((f \alpha g^m) g^m) g^m \\
(\varpi \circ K_{\mathcal{F}}(\alpha))_{(g^m)} &= \bigvee_{g^m = ((f \alpha g^m) g^m) g^m} \{\varpi((f \alpha g^m) g^m) \wedge K_{\mathcal{F}}(\alpha)(g^m)\} \\
&\geq \varpi(g^m) \wedge K_{\mathcal{F}}(\alpha)(g^m) \\
&\geq \varpi(g^m) \wedge 1 \\
&= \varpi(g^m)
\end{aligned}$$

$\varpi \circ K_{\mathcal{F}}(\alpha) \geq \varpi$  and

$\varpi \circ K_{\mathcal{F}}(\alpha) \leq \varpi$  [because  $\varpi$  is fuzzy-two sided ideal of  $\mathcal{F}$ ]

Therefore  $(\varpi \circ K_{\mathcal{F}}(\alpha)) = \varpi$ .

**Corollary (1. 9)**

In Semi-weakly  $m$  regular LAsemi-g  $\mathcal{F}$ ,  $(K_{\mathcal{F}}(\alpha) \circ K_{\mathcal{F}}(\alpha))=K_{\mathcal{F}}(\alpha)$ .

**Proposition(1. 10)**

A fuzzy-subset  $\varpi$  of Semi-weakly  $m$  regular LAsemi-g  $\mathcal{F}$  with left identity be the fuzzy-left ideals in  $\mathcal{F}$  iff it is the fuzzy-right ideals for the  $\mathcal{F}$  [12], [13].

**proof**

$\Rightarrow$  let  $\mathcal{G}_1, \mathcal{G}_2 \in \mathcal{F}$  for some  $\alpha_1, \alpha_2, f_1$  and  $f_2 \in \mathcal{F}$

s.t  $\mathcal{G}_1^m = \mathcal{G}_1^m \alpha_1 \mathcal{G}_1^{2m} f_1$  and

$\mathcal{G}_2^m = \mathcal{G}_2^m \alpha_2 \mathcal{G}_2^{2m} f_2$ , where  $m$  is constant.

$$\begin{aligned} \mathcal{G}_1^m \mathcal{G}_2^m &= \mathcal{G}_1^m \alpha_1 \mathcal{G}_1^{2m} f_1 \mathcal{G}_2^m \\ &= (\mathcal{G}_1^m (\alpha_1 \mathcal{G}_1^{2m} f_1)) \mathcal{G}_2^m \text{ [by the left invertive law[3]]} \\ &= (\mathcal{G}_2^m (\alpha_1 \mathcal{G}_1^{2m} f_1)) \mathcal{G}_1^m. \end{aligned}$$

$$\begin{aligned} \varpi (\mathcal{G}_1^m \mathcal{G}_2^m) &= \varpi ((\mathcal{G}_2^m (\alpha_1 \mathcal{G}_1^{2m} f_1)) \mathcal{G}_1^m) \\ &\geq \varpi (\mathcal{G}_1^m) \text{ [because } \varpi \text{ is fuzzy-left ideal]} \end{aligned}$$

$$\varpi (\mathcal{G}_1^m \mathcal{G}_2^m) \geq \varpi (\mathcal{G}_1^m),$$

hence  $\varpi$  is fuzzy-right ideals.

$\Leftarrow$  Let  $\varpi$  is a fuzzy-right ideals for the

Semi-weakly  $m$  regular LAsemi-g  $\mathcal{F}$  with left identity,

$$\begin{aligned} \mathcal{G}_1^m \mathcal{G}_2^m &= \mathcal{G}_1^m \alpha_1 \mathcal{G}_1^{2m} f_1 \mathcal{G}_2^m \\ &= (\mathcal{G}_1^m (\alpha_1 \mathcal{G}_1^{2m} f_1)) \mathcal{G}_2^m \text{ [ by the left invertive law[3]]} \\ &= (\mathcal{G}_2^m (\alpha_1 \mathcal{G}_1^{2m} f_1)) \mathcal{G}_1^m. \end{aligned}$$

$$\begin{aligned} \varpi (\mathcal{G}_1^m \mathcal{G}_2^m) &= \varpi ((\mathcal{G}_2^m (\alpha_1 \mathcal{G}_1^{2m} f_1)) \mathcal{G}_1^m) \\ &\geq \varpi (\mathcal{G}_2^m) \text{ [because } \varpi \text{ is fuzzy-right ideal]} \end{aligned}$$

$$\varpi (\mathcal{G}_1^m \mathcal{G}_2^m) \geq \varpi (\mathcal{G}_2^m).$$

Hence  $\varpi$  is fuzzy-left ideal.

**Proposition(1. 11)**

If fuzzy-subset  $\varpi$  of Semi-weakly  $m$  regular LAsemi-gr  $\mathcal{F}$  with left identity then  $\varpi$  is fuzzy-(1,2) ideals for the  $\mathcal{F}$  iff it is fuzzy-two side ideals for the  $\mathcal{F}$ .

**proof**

$\Rightarrow$  assume  $\varpi$  be fuzzy-(1, 2) ideals for the Semi-weakly  $m$  regular LAsemi-g  $\mathcal{F}$  and let  $u, z \in \mathcal{F}$  for some  $v, w \in \mathcal{F}$  s.t

$u^m = u^m v u^{2m} w$ , where  $m$  is constant.

$$\begin{aligned} z u^m &= z u^m v u^{2m} w \\ &= ((z u^m) v u^{2m} w) \text{ so we have:} \\ &= u^m (z v u^{2m} w) \text{ [by the medial law [3]]} \\ &= u^m ((z u^{2m})(v w)) \\ &= u^m (u^{2m} (z v w)) \text{ [by the left invertive law[3]]} \\ &= u^m (v w z) u^{2m} \text{ [by the left invertive law[3]]} \end{aligned}$$

$$\begin{aligned} \varpi (z u^m) &= \varpi (u^m (v w z) u^{2m}) \\ &= \varpi (u^m (v w z) u^m u^m) \\ &\geq \varpi (u^m) \wedge \varpi (u^m) \wedge \varpi (u^m) \text{ [because } \varpi \text{ is a fuzzy-(1, 2) ideal]} \end{aligned}$$

$$\varpi (z u^m) \geq \varpi (u^m),$$

then  $\varpi$  is fuzzy-left ideal and by proposition (1. 10)  $\varpi$  is fuzzy right ideals for the  $\mathcal{F}$ .

$\Leftarrow$  Let  $\varpi$  be the fuzzy two side ideals for

the  $\mathcal{F}$  Semi-weakly  $m$  regular LAsemi-g, with left identity, and

let  $u_1, u_2, u_3$  and  $z \in \mathcal{F}$ .

$$\begin{aligned} \varpi (u_1^m z) (u_2^m u_3^m) &\geq \varpi (u_1^m z) \text{ [because } \varpi \text{ is fuzzy-right ideal]} \\ &\geq \varpi (u_1^m) \text{ [ because } \varpi \text{ is fuzzy-right ideal]} \end{aligned}$$

$$\begin{aligned} \varpi (u_1^m z) (u_2^m u_3^m) &\geq \varpi (u_2^m u_3^m) \text{ [ because } \varpi \text{ is fuzzy-left ideal]} \\ &\geq \varpi (u_2^m) \text{ [ because } \varpi \text{ is fuzzy-right ideal]} \end{aligned}$$

$$\begin{aligned} \varpi (u_1^m z) (u_2^m u_3^m) &\geq \varpi (u_2^m u_3^m) \text{ [ because } \varpi \text{ is fuzzy-left ideal]} \\ &\geq \varpi (u_3^m) \text{ [ because } \varpi \text{ is fuzzy-left ideal]} \end{aligned}$$

$$\varpi (u_1^m z) (u_2^m u_3^m) \geq \varpi (u_1^m) \wedge \varpi (u_2^m) \wedge \varpi (u_3^m)$$

This shown  $\varpi$  be the fuzzy-(1,2) ideals

**Proposition(1. 12)**

In Semi-weakly  $m$  regular LAsemi-g  $\mathcal{F}$  with left identity then  $\varpi$  be the fuzzy (1,2) ideals for the  $\mathcal{F}$  iff  $\varpi$  be the fuzzy quasi ideals for the  $\mathcal{F}$  [14].

**proof**

$\Rightarrow$  by proposition (1. 11) and proposition (1. 8) is an easy consequence.

$\Leftarrow$  Let  $\varpi$  be the fuzzy-quasi ideals for the Semi-weakly  $m$  regulars LAsemi-g  $\mathcal{F}$ , with the left identities ,

assume that  $u \in \mathcal{F}$  for some  $v, w \in \mathcal{F}$  s.t  $u^m = u^m v u^{2m} w$ , where  $m$  is constant.

$$u^m = u^m v u^{2m} w$$

$$= u^m [(v u^{2m}) w e] \quad [\text{the left invertive law}[3]]$$

$$= u^m [e w (v u^m u^m)]$$

$$(\varpi \circ K_{\mathcal{F}})_{(u^m)} = \bigvee_{u^m = u^m [e w (v u^m u^m)]} \{ \varpi(u^m) \wedge K_{\mathcal{F}}(v) (e w v u^m u^m) \}$$

$$\geq \varpi(u^m) \wedge 1$$

$$= \varpi(u^m)$$

Now, by using proposition (1. 7) we get

$$(K_{\mathcal{F}} \circ \varpi) = \varpi \text{ and corollary (1,9) we get}$$

$$(K_{\mathcal{F}} \circ K_{\mathcal{F}}) = K_{\mathcal{F}}.$$

$$\text{Therefore } (\varpi \circ K_{\mathcal{F}}) = (K_{\mathcal{F}} \circ \varpi) \circ (K_{\mathcal{F}} \circ K_{\mathcal{F}}) \circ (\varpi \circ K_{\mathcal{F}})$$

$$= (K_{\mathcal{F}} \circ K_{\mathcal{F}}) \circ (\varpi \circ K_{\mathcal{F}})$$

$$= (K_{\mathcal{F}} \circ (\varpi \circ K_{\mathcal{F}})) \supseteq (K_{\mathcal{F}} \circ \varpi).$$

$$\text{Hence } (K_{\mathcal{F}} \circ \varpi) \subseteq (K_{\mathcal{F}} \circ \varpi) \cap (\varpi \circ K_{\mathcal{F}}).$$

As  $\varpi$  is a fuzzy-quasi ideal of  $\mathcal{F}$ ,  $(K_{\mathcal{F}} \circ \varpi) \subseteq \varpi$ .

Now lemma(1. 2. 9)[10] and proposition (1. 11),

then we get  $\varpi$  be the fuzzy two side ideals and from proposition (1. 11),

we get  $\varpi$  is a fuzzy-(1, 2) ideal [15].

### 3. Conclusion

A semi-g  $\mathcal{F}$  is called Semi-weakly  $m$  regular if for all  $\tau \in \mathcal{F}$  there exists  $v, w \in \mathcal{F}$  s.t  $\tau^m = \tau^m v \tau^{2m} w$ , where  $m$  is constant.

We concluded that:

1. If  $\varpi$  is a fuzzy-interior ideal in Semi-weakly  $m$  regular LAsemi-g  $\mathcal{F}$ , with identity, then  $\varpi(\alpha^m \dagger^m) = \varpi(\dagger^m \alpha^m)$  for all  $\alpha, \dagger \in \mathcal{F}$ ,  $m$  is a constant.
2. For a fuzzy-subset  $\varpi$  of Semi-weakly  $m$  regular LAsemi-g  $\mathcal{F}$  and  $K$  is a subset of  $\mathcal{F}$ , then  $(K_{\mathcal{F}} \circ \varpi) = \varpi$ .
3. In Semi-weakly  $m$  regular semi-g  $\mathcal{F}$ ,  $(K_{\mathcal{F}}(\alpha) \circ \varpi) = \varpi$  and  $(\varpi \circ K_{\mathcal{F}}(\alpha)) = \varpi$  holds for every fuzzy two side ideals  $\varpi$  in  $\mathcal{F}$ .
4. The fuzzy-subset  $\varpi$  of Semi-weakly  $m$  regular LAsemi-g  $\mathcal{F}$  with left identity be the fuzzy-left ideals in  $\mathcal{F}$  iff it is the fuzzy-right ideals for the  $\mathcal{F}$ .
5. In Semi-weakly  $m$  regular LAsemi-g  $\mathcal{F}$  with left identity then  $\varpi$  be the fuzzy (1,2) ideals for the  $\mathcal{F}$  iff  $\varpi$  be the fuzzy quasi ideals for the  $\mathcal{F}$ .

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