CENTRAL ASIAN JOURNAL OF MATHEMATICAL THEORY AND COMPUTER SCIENCES
https://cajmtcs.centralasianstudies.org/index.php/CAJMTCS
Volume: 07 Issue: 02 | April 2026 ISSN: 2660-5309

.

Article

Using Definite Integrals to Estimate Curved Areas and Volumes of Domes and Architectural Structures

Merouge N. Kadhem*[1]

1. Karbala Directorate of Education, Ministry of Education, Iraq.
* Correspondence: merougewizma1976@gmail.com

Abstract: Curved architectural forms mark the finest achievement of structural engineering in architecture. For centuries, the design is one of many that continues to admire contemporary architectural history. Their structural and spatial efficiency has been a journey through domes, arches, and vaults. In architectural planning, structural analysis and estimating the cost of materials, accurate estimation of surface areas and volumes is very important. Geometric formula alone suffices to give an exact solution of simple geometric shapes only like a sphere or cylinder. But most building structure has complex curves that requires math modeling. The definite integral is a mathematical instrument for estimating the area and volume of structures with irregular curves. The use of definite integrals to find the curved surface areas and volumes of domes in architecture is examined in this study. This paper provides a summary of integral mathematics, including the definite integral, proofs and applications for architectural geometry. Spherical domes, parabolic domes and elliptical domes are some examples of dome structures whose mathematical models have all been studied. Other topics covered are the disk method, shell method, and surface of revolution for volumes and surface areas of solids of revolution. Instances from both historical and contemporary architecture are offered to show how calculus based estimation can assist engineering choices in architectural design. A series of practical numerical approximations have also been presented to indicate that definite integrals can be used in construction. The definite integral is thus an important connection between mathematics and architecture, allowing for the ability to estimate areas and volumes of more complex curved structures. These techniques assist designers and engineers to develop cost-effective systems, maximize resource utilization, and advance structural modelling.

Keywords: Definite integrals, Architectural geometry, Dome structures, Curved surface estimation, Volume calculation.

Citation: Kadhem M. N. Using Definite Integrals to Estimate Curved Areas and Volumes of Domes and Architectural Structures. Central Asian Journal of Mathematical Theory and Computer Sciences 2026, 7(2), 238-250.

Received: 10th Jan 2025
Revised: 11th Feb 2026
Accepted: 15th Mar 2026
Published: 05th Apr 2026

1. Introduction

Mathematics is used in one form or another in the design of an architectural or structural engineer. Many buildings require precise estimation of curved surface geometry in their planning and construction stages. The dome is one of the most remarkable particular types of curved structures. You can see such structures in the churches, temples, historic places, museums, and modern public buildings. To determine the materials, structural properties and minimize the cost of construction, engineers require the accurate numerical measure of the surface areas and volume.

Well-developed mathematical formulas can provide an easy solution for together with the volume of simple shapes like cylinders, cones, or spheres. However, a number of architectural domes incorporate more complex curves as well as curves which require advanced geometry to explain. In these situations, calculus becomes necessary in mathematics. The definite integral is a useful tool for curved geometries because engineers deal with quantities that vary smoothly with one another.

Sometimes, a dome can be approximated as a surface of revolution of a curve about an axis. In these situations, a mathematical function that gives the profile of the structure defines it very well. The volume delimitated by the structure and the area of its curved surface can be calculated using integration methods. These calculations are important for design, measurements, and analysis of structures.

Many great civilizations designed fantastic domes before calculus! Built structures such as the Pantheon at Rome, Byzantine church and Islamic Mosque believed building set up on earth through Geometric Approximation and Empirical Knowledge Nowadays mathematical methods of communicative power far beyond what any architect of the pyramids or their successors could have conceived of are used to analyses such structures with great accuracy.

But calculus based mathematical models are still used in current architecture (A little caveat: Definite Integrals becomes more relevant in architecture with current methods of Computational design) This is the option that allows architect to consider these surfaces as it enables you to simulate complex surfaces with very simple geometric characteristics.

Curved surfaces Unlike a typical wall Assignment of structural load: dome Compression Load Physics Mathematically modeling these loads some precision, the engineers can estimate precisely what masses these structures will frequently be subjected to and design them to stand firm in even the conditions.

Furthermore, both acoustical performance and sustainability would be affected as it lights distribution and a myriad of other processes as architectural surfaces are rarely straight. The big domes have some interesting reflections but the curvature also plays a role in how much light the building can get through the dome. With the known mathematics geometry of these surfaces architects plan structures to be very effective. [1]

Thus, we see how definite integrals bridge the gap between mathematics and engineering practice at least in architectural geometry. Mathematical models provide an efficient and reliable approach to estimate the geometric features of curved structures. These models are used for moving a design from the drawing board to the construction site and these aids to the architect.

1.1 Research Background

Particularly, curved structures were the most fundamental part of architecture for a long time. Dome, arch, and vault are elements on ancient buildings that quality large interior spaces with few or no supporting columns. These provided a structural economy and a visual appeal.

Construction workers used geometrical approximations to measure structures before to calculus. Proportional measurements for design were based on experience and using physical measurements during building. These techniques enabled impressive structures to be built, but did not always represent accurate geometric calculations.

During the seventeenth century the creation of calculus provided a tool for investigating curved shapes that was far more powerful than anything seen until that point. For example, definite integrals allowed mathematicians to find the exact area that a curve described and compute volumes generated by rotating curves about an axis. These ideas became very important in engineering and physics.

Later on, these methods from mathematics found a way into architectural engineering for complex structure analysis. If a dome profile could be defined by a mathematical function, it can be integrated in order to find the geometrical properties of such a resultant structure. This enables the engineer to accurately judge structural volumes, material requirement and surface areas.

Architectural design has become computationally modeled more recently. The digital design platforms rely on mathematical algorithms (based on integration methods) that are able to perform this task. These systems autonomously provide three-dimensional modelling of continuous shapes and automatically conduct analysis of their physical characteristics.

So, Mathematics and architecture have thus become one in one of the fundamental building design elements of the time. In order for architects and engineers to come up with innovative designs, they must understand the math behind these calculations in order to keep these structures safe and esthetically efficient.

1.2 Research Problem

We encounter curved surfaces in many architectural structures that cannot be studied with simple geometric formulas. Curved structures for instance many domes, shells and vaulted roofs have complicated curves and subsequent geometry that requires detailed mathematical modeling.

In this process of estimating materials, evaluating structural loads and calculating construction costs, architects and engineers must find the volumes and surface areas of these structures. These are traditional geometric approaches where those may give wrong results when dealing with irregular or non-standard curved shapes.

This research is solving the main problem of the mathematical estimation of architectural areas and volumes of curved shapes of the domes. This makes obtaining accurate geometric properties for these structures challenging without the proper mathematical toolbox.

This is the type of problem which can be solved using a mathematical approach called definite integrals. It must be noted that there are a number of architectural studies, focused on structural design but not on the mathematical modeling techniques used to extract geometric measurements.

Hence this research is focused on using definite integration to develop models that can estimate the curved surface area and the corresponding volume of given domes as well as to calculate the aesthetic designs of the same and other buildings. Using mathematical modeling to support architectural engineering with accurate geometric calculations is the goal of the study.

1.3 Research Objectives

This research aims to explore the role of definite integrals in estimating curved areas and volumes of architectural structures. The objectives of the study include the following.

- Describe the reflection of definite integration within geometric measurement in mathematics.
- Model curvilinear architectures based on mathematical functions
- Show an application of integration in finding the volumes of dome-type structures.
- Discuss techniques for calculating curved surface areas created when curves are rotated.
- Provide realistic mathematical models for typical dome shapes of varying geometry such as spherical, parabolic and elliptical domes.
- Emphasizing on Mathematical modelling in architectural engineering and design.

1.4 Research Significance

Many architects and engineers use tools that require exact meat on the bones of curves that only definite integrals provide. By taking these measurements, we can determine some of the most critical parameters of construction and design.

Understanding modelling volume enables the tooling engineers to measure the amount of materials required to construct a dome. When it comes to budgeting, resource allocation, and planning, this type of data goes a long way.

Surface area calculations come handy too when you are to determine how much finishing material (such as tiles, glass panels, or metal sheets) to cover these curved structures. It is also a key to minimize the wastage and increase the efficiency in the construction site.

Structural — Algorithms that predict how loads inequity in time on complex, curved surfaces are very helpful to engineers. Additionally, because the path of compression forces follows a developable curve, the compression forces which provide stability for a dome are aided by this property.

And opened up also, new designs of architecture, mathematical analysis also gives us. It fights the natural tendency to be governed by existing constructions and codes, and allowing architects to establish what they want to build geometrically, and therefore offers them new forms and structural possibilities.

This study effectively conveys the function of calculus relative to the architectural engineering discipline while demonstrating an incremental (uninterrupted set of numerical steps) calculus solution approach for modelling and interposing curves in vertical or horizontal design and analysis of structural-object-based systems.

2. Literature Review

Math has been part of architecture for centuries, for example in curvilinear shell structure analysis. Archetypal orbs of veritable architect states of mind met upon the basics of geometry that at most only hint at the curvature of motion. However, calculus provided new tools for computing the areas of more irregular shapes.

Definite integrals provide a very useful way for us to find areas and volumes that are inside regions involving continuous functions. Application to Geometry: Integration makes it possible to specify the area enclosed by a curve and the volume of the solid created by the spinning of a curve about an axis. These principles were established for analyzing curved work. [2]

This method of classification is essentially built on the method already proposed by classical mathematicians. As one example, Archimedes established geometric techniques for approximating area and volume. His way of exhaustion is to find bound areas between curves by intersecting them with a sequence of figures whose area can be computed. Calculus itself was thousands of years younger than this procedure for calculating an area under a curve, and yet implicitly was responsible for the flavor of summing infinitesimals.

The formal treatment of integral calculus by newton and Leibniz changed the game of mathematical analysis. Afterwards, they proved that the definite integral is a useful tool to compute these kinds of sums of the infinitesimals. These methods are subsequently utilized by engineers for physical system modeling and geometrical formulation.

Integration as the Key Idea of Geometric Measurement: From Leibniz's Day Up to Now While integration is one of the important ideas of calculus which forms the fundamental theorem of calculus, modern calculus textbooks also emphasize the central role of integration on geometric measurements. For instance, some definite integrals provide the formulas of volumes of solids of rotation, or the area of surfaces of solids produced by turning a curve. These concepts carry over really well to domes or other kinds of buildings.

It has been shown by architectural geometers that most domes can be represented by mathematical functions. They deliver in circular (spherical domes, 2D circular equations), parabolic domes (2D quadratic equations), or elliptical domes (2D conic-section equations) shapes. Such mathematical abstractions thus provide a direct basis upon which one is able to specifically target architectural design for integration techniques.

So much so, that architectural historians have too sought out geometry to help describe the basic principles of how one has built a dome throughout history. The catch two ends or unique game plans of focuses on a vault where it follows recurred bends that stream heaps in the most productive way. Mathematically modeling these structures has enabled researchers to study their characteristics and identify the basic engineering principles based on which they were designed.

Computational architecture brought mathematical concepts closer to architectural design. In most digital modeling tools, all curved surfaces are generally defined mathematically by parametric equations. These systems leverage numerical integration algorithms to derive geometric characteristics of arbitrary forms.

Rather than being an illustration of certain relationships, parametric architecture is built on mathematical relationships as a means to define form. Architects may use parameterization of a mathematical equation to create an output of multiple design options with their geometry maintained. [3]

Architectural analysis also relies heavily on numerical integration methods. If the exact physical function governing a structure is unknown, the geometry can be approximated with discrete data points, which is often how experimental data will be reported. Numerically estimating areas and volumes is achieved with methods such as the trapezoidal rule and Simpson's rule.

Mathematical modeling is often used in the climate of shell structures and dome roofs in structural engineering studies. The geometry of arches directs the officer to compression as the force adapts to the geometry of the structure Almost all arches mediate loads in a similar method, simply they all distribute to curved areas, meaning compressive forces will not travel straight through the arch just as they would through a beam. Engineers can assess these load paths, and determine structural capacity, using integration techniques.

Emerging kinds of architecture emerge from a study of math which, while clearly driven by the technological application of math to our current digital tools, still has architectural ideas that revel in a specificity to the math but bypass any structural consequence. Through the use of computer-based methods, architects are able to replicate the behavior of certain objects, reduce the amount of materials used, and explore specific qualities of high complex geometries.

Almost in the entire review contained in this section, integration is one of the mathematical tools for architectural engineering that can be categorized as something basic and most fundamental. From the area and the volume estimation from classical geometry all up to modern computational design: definite integrals as the cornerstone [4]

Methodology

In the present study, estimation of curved surface areas and volumes of dome-shaped architectural structures were carried out using the definite integrals through analytical mathematical modeling based approach. Geometries were parameterized by mathematical functions that provided a cross-sectional representation of a dome, which were then revolved around an axis to form 3D models. Dome shapes including spherical, parabolic, and elliptical forms were chosen as archetypical related structural types as they were found to be common as structural forms of both historical as well as current designs. The expression of each dome profile as a continuous function enabled us to use integral calculus to determine geometric properties. Using integration techniques such as the disk method and the cylindrical shell method the volume of these structures were calculated by dividing the dome into infinitisimal elements and summing the total volume through definite integration. We estimated the surface area, taking curvature into account by making use of the surface of revolution formula and using the derivative of the defining function. If explicit mathematical functions were not found, numerical integration methods including the trapezoidal rule and Simpson's rule then applied to computing approximate results from non-continuous data points obtained from architectural measurements or models. Including practical numerical examples, to illustrate the applicability of such methods for architectural practice, specifically in relation to material quantity and structural capacity estimations. This methodological approach allows us to systematically and precisely analyze complex curved structures and enables mathematical modeling, while offering inherent support for architectural design, planning, and engineering analysis.

Result and discussion

3. Mathematical Methodology and Integral Modeling of Dome Structures

In architectural structures, the estimation of the curved areas and volumes recovery requires either mathematical models which describe the geometry of the structure. Definite integrals give engineers and architects the ability to measure how much there is of something for a curved object with a well-defined mathematical description. Many there where in turn cross-section of Domes, arches and vaulted roofs can be represented by mathematical functions. Now that the mathematical function has been found, integration techniques can be employed to calculate the area, volume, and surface properties of the structure. [5]

Dome in Architectural Analysis form is often a surface of revolution. This forms a two-dimensional curve sweeping around some axis, and creates a surface of revolution. The three-dimensional shape resulting from this method produces a continuous curve surface. This geometric rule describes the principle of many architectural domes as it gives a structure that is both stable and aesthetically pleasing when a symmetrical curve is rotated around a vertical axis.

In order to formulate the analysis of such structures mathematically, the curve through its cross-section needs to be defined as a general function. Let C be a curve given by the equation

$$y = f(x)$$

where x represents the horizontal coordinate and y represents the height of the dome profile. If this curve is rotated around the x-axis or the y-axis, a three-dimensional surface is formed. Definite integrals can then be used to compute the volume enclosed by this surface.

3.1 Volume Estimation Using the Disk Method

One of the most common techniques for determining the volume of a rotational solid is the disk method. This method divides the structure into many thin circular disks stacked along the axis of rotation. Each disk represents a small cross-sectional volume. By summing the volumes of all disks through integration, the total volume of the structure can be determined. [6]

If a function $y = f(x)$ is rotated around the x-axis between $x = a$ and $x = b$, the volume of the resulting solid can be expressed as

$$V = \pi \int_a^b [f(x)]^2 \, dx$$

This equation shows that the volume is obtained by integrating the square of the radius of each disk along the interval. The radius corresponds to the value of the function $f(x)$.

In architectural applications, this method is particularly useful when modeling domes that follow a symmetrical curve. For example, if the cross-section of a dome follows a semicircular profile, the function describing the curve can be written as

$$y = \sqrt{R^2 - x^2}$$

Here, R is the radius of the sphere. When this semicircle is rotated around x-axis we will get a continuous structure which will be hemisphere. Volume of the hemispherical dome: using nick for applying the disk method.

From -R to R and integrating the function gives you the volume of the sphere. The volume of the hemispherical dome can be found by dividing the result by two.

Integration like this illustrates how integration makes a geometric problem a math problem.

3.2 Volume Estimation Using the Shell Method

Another integration technique used in geometric analysis is the cylindrical shell method. This method is particularly useful when the axis of rotation is parallel to the axis of the function. Instead of disks, the structure is divided into thin cylindrical shells. [7]

If a function $y = f(x)$ is rotated around the y-axis, the volume of the resulting structure can be calculated using the formula

$$V = 2\pi \int_a^b x\, f(x) dx$$

Here, x is the radius of the shell and f\left(x\right) is the height of the shell. A numerical method, an example of which is used in preview, is employed to add up the volumes of each of the shells to arrive at the total volume of the structure.

The shell method excels in architectural modeling since many domes used in architecture are defined naturally by vertical profiles. Rotating the profile curve produces a simple contour for applying the shell method often making observations that the disk method is more cumbersome.

3.3 Surface Area of Rotational Structures

In architectural engineering, calculating the surface area of a dome is often as important as determining its volume. Surface area estimation is necessary for calculating the amount of construction material required to cover the structure.

If a curve $y = f(x)$ is rotated around the x-axis, the surface area of the resulting structure can be determined using the following integral [8]

$$S = 2\pi \int_a^b f\,(x)\sqrt{1 + [f'(x)]^2} dx$$

The term $f'(x)$ represents the derivative of the function. This derivative accounts for the slope of the curve. The square root component adjusts the surface measurement to reflect the curvature of the structure.

This formula enables accurate measurement of curved surfaces that cannot be measured using simple geometric formulas.

For example, if a dome follows a parabolic curve such as

$$y = ax^2 + bx + c$$

the derivative of the function becomes

$$f'(x) = 2ax + b$$

Substituting this derivative into the surface area equation allows the calculation of the curved surface area of the dome.

3.4 Mathematical Modeling of Common Dome Shapes

Architectural domes typically follow several common geometric patterns. Each pattern can be modeled using a mathematical function. Integration techniques can then be applied to estimate the geometric properties of the structure. [9]

Spherical Dome Model

Spherical domes represent one of the oldest architectural forms. The cross-section of a spherical dome follows the equation of a circle

$$x^2 + y^2 = R^2$$

Solving for y produces the function

$$y = \sqrt{R^2 - x^2}$$

Rotating this curve around the vertical axis generates a spherical dome.

The volume of a spherical dome can be calculated by applying the disk method. The surface area can be determined using the surface-of-revolution formula.

Parabolic Dome Model

Parabolic domes are widely used in modern architecture because they distribute structural loads efficiently. The cross-section of a parabolic dome can be expressed as

$$y = kx^2$$

where k determines the curvature of the dome.

When this curve is rotated around the vertical axis, the resulting structure forms a paraboloid. Definite integrals can be used to calculate both the volume and surface area of the paraboloid.

Elliptical Dome Model

Elliptical domes provide greater vertical height and are commonly used in classical architecture. The equation of an ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Solving for y produces

$$y = b\sqrt{1 - \frac{x^2}{a^2}}$$

Rotating this curve around the vertical axis generates an elliptical dome. Integration techniques allow the calculation of its geometric properties.

3.5 Numerical Integration in Architectural Practice

In real-life constructions, the mathematical function describing a dome may not be available. Instead, the engineers scanning the profile of the structure at multiple points. You can then apply numerical integration methods to compute the area or volume.

The trapezoidal rule and Simpson's rule are two examples of these numerical methods.

The area under the curve is approximated in trapezoids. By adding all these trapezoids, we get a value of the integral.

Speaking of approximation, Simpson's rule is a little better than the above due to approximating the curve using quadratic polynomials. When the function is smooth this method gives very accurate results.

In modern day architecture software, these calculations are done automatically using numerical integration algorithms. Nonetheless, the basis of calculation is still the definite integral.

By having this knowledge of mathematical theory, the architects and engineers can analyze again the computational outputs more closely and can design more effective structures. [10]

4. Applications of Definite Integrals in Architectural Dome Structures

As this is more common in architectural engineering of dealing with structural elements that are obviously curved, so it is apparent that definite integrals cannot be avoided. The geometric property should be determined precisely since the structural safety and efficiency of the dome, vaulted roof, and other shells and curved facet forms are dependent on the architectural technique used in such shell structures. In lots of the architectural projects these complete externs that molds either well could not talked in direct mathematical words them required complex formulas and stunning fabrication ways to make sure the items designed in software will construct in practical. Clearly there are plenty geometric shapes that can be defined mathematically and you surely can trust the mathematical integral to assuredly get the volume (and surface area) of most of them.

Domes, architectural types of domes are often modeled forms of surfaces generated by revolving a curve around an vertical axis. This gives engineers the ability to mathematically model the structure, and then simply use integrations to find its geometrical properties. Using the dome profile/contour in the vertical plane as a function, definite integrals relate the volume under the dome and the area of its trend surface. [11]

One of the practical uses of definite integrals in architecture can be observed in several types of dome: a spherical, parabolic, and elliptical. Each of those forms has its own geometry, which controls not only structural behavior but architectural appearance.

4.1 Volume Estimation of a Spherical Dome

Spherical domes represent one of the most common dome structures used throughout architectural history. Many historic monuments use hemispherical domes because they provide structural strength and symmetrical geometry. [12]

The cross section of a spherical dome follows the equation of a circle

$$x^2 + y^2 = R^2$$

Solving for y

$$y = \sqrt{R^2 - x^2}$$

When this curve rotates around the horizontal axis it forms a sphere. A hemispherical dome corresponds to half of this sphere.

The volume of the sphere can be obtained using the disk integration method

$$V = \pi \int_{-R}^{R} (R^2 - x^2)\, dx$$

Evaluating the integral

$V = \pi \left[R^2 x - \frac{x^3}{3}\right]_{-R}^{R}$ $V = \frac{4}{3}\pi R^3$

Since the dome represents half of the sphere the dome volume becomes

$$V = \frac{2}{3}\pi R^3$$

Practical architectural example

Assume a hemispherical dome with a radius of 12 meters.

$V = \frac{2}{3}\pi(12)^3$ $V = \frac{2}{3}\pi(1728)$ $V \approx 3619.1$ cubic meters

This calculation provides architects with the internal capacity of the dome. Engineers can use this value to estimate air volume, acoustic behavior, and interior spatial capacity.

4.2 Surface Area Calculation of a Dome

Surface area estimation is essential when determining the amount of material required for dome construction. Architects must know how much concrete, steel panels, glass panels, or tiles are needed to cover the curved structure.

The surface area of a curve rotated around the x axis can be calculated using the surface of revolution formula [13]

$$S = 2\pi \int_a^b y \sqrt{1 + (y')^2} dx$$

For a spherical dome

$$y = \sqrt{R^2 - x^2}$$

After differentiation

$$y' = \frac{-x}{\sqrt{R^2 - x^2}}$$

Substituting into the surface area equation and evaluating the integral results in the well known formula

$$S = 2\pi R^2$$

Architectural interpretation

For a dome with radius 12 meters

$S = 2\pi(12)^2$ $S = 2\pi(144)$ $S \approx 904.78$ square meters

This value represents the external surface area of the dome. If tiles measuring 0.25 m² are used for cladding the dome then the approximate number of tiles required is

$$\frac{904.78}{0.25} \approx 3619 \text{ tiles}$$

Such calculations are essential for cost estimation and material procurement.

4.3 Modeling a Parabolic Dome Using Integration

Modern architecture frequently uses parabolic domes because they distribute structural loads efficiently. The cross section of a parabolic dome can be represented by the quadratic equation [14]

$$y = ax^2$$

Assume a dome described by

$$y = 0.05x^2$$

with the base extending from $x = -10$ to $x = 10$.

When this curve rotates around the vertical axis the dome forms a paraboloid structure.

The volume of the structure can be calculated using the shell method

$V = 2\pi \int_0^{10} x\,(0.05x^2)dx$ $V = 0.1\pi \int_0^{10} x^3\,dx$ $V = 0.1\pi \left[\frac{x^4}{4}\right]_0^{10}$ $V = 0.1\pi(2500)$ $V = 250\pi$ $V \approx 785.4$ cubic meters

Parabolic domes appear in stadium roofs, exhibition halls, and modern public buildings because they provide excellent structural efficiency.

4.4 Elliptical Dome Structures

Elliptical domes represent another common architectural form. These domes are often taller than spherical domes while maintaining structural stability.

The equation of an ellipse is [15]

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Solving for y

$$y = b\sqrt{1 - \frac{x^2}{a^2}}$$

If we were to take the curve and spin it around the vertical axis, we would get an elliptical dome.

This allows us to obtain an integral for the volume of the object. It is a much more involved calculation than the spherical case but will follow the same kind of math. Since these integrals just are often too complicated to compute analytically, architects implement numerical tools as a way to compute it numerically.

This is a common elliptical dome shape found in classical buildings, because it allows much taller spaces overhead without great increases in structural load.

4.5 Numerical Integration in Architectural Measurement

Almost none of the real construction projects that use architects get to work with such clean full-rank access to such nice mathematical functions for dome profiles as in the image, where faces with less than full access to mathematical form are actually an ideal geometric object flattened by the application of mathematical formulae. Engineers gather data points from architectural blueprints or laser scanning systems and input them into the software, rather than issuing measurements and calculations by hand. To do so numerical integration methods are then used to provide an estimate of the area and volume of such structure.

Two commonly used techniques are

Trapezoidal rule

Simpson rule

Trapezoidal Approach: The second technique of finding out area under the curve is Trapezoidal approach, where area is divided in to trapezoids. The formula for trapezoidal rule is:

$$\int_a^b f(x)dx \approx \frac{h}{2}[f(x_0) + 2f(x_1) + \ldots + f(x_n)]$$

Simpson rule improves accuracy by approximating the curve using quadratic polynomials

$$\int_a^b f(x)dx \approx \frac{h}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + \ldots + f(x_n)]$$

These numerical methods are widely implemented in architectural design software.

4.6 Architectural and Engineering Implications

A Few Practical Benefits of Using Definite Integrals to Design Domes

Accurate estimation of material quantities

Improved structural analysis

A better estimation of the cost of construction behind the planning

Higher abilities in structures of advanced curvilinear geometries

Combined with integration methods, this provides architects the opportunity to experiment with new geometries. Calculus is often needed to calculate shapes for architectural free form domes, shell roofs, and organic forms, so parameterizing those shapes is inherently difficult.

Many recent computational design platforms now automatically leverage integration algorithms whenever geometric properties for architectural models are computed. However, aside from that, the basics — i.e. the math's behind this specific research – equip architects and engineers with the tools needed to perceive these results, without having to consider them as true gospel.

And so it is these definite integrals that help even up mythical abstractions of maths to the realist designs of architectural engineering. With mathematical modeling engineers can design structures that are beautiful to look at and at the same time structurally efficient.

5. Conclusion and Future Work

Mathematics in architecture — the importance of curved shapes in architecture design and assessment Dome, Arch, and Shell Roofs — Geometric estimation of the surface of the entire architectural structures Real-life structures are not easily represented with simple geometric volume & area formula due to the curved surfaces involved. The type and techniques can only be expressed by the particular mathematical techniques such as definite integrals for such complicated geometries.

In this study we derived definite integrals for both volumes and surface area for a class of dome shapes. Dome profiles are normally given by an algebraic function which enables the normal geometric properties to be quickly integrated, and all of these can be useful to engineers. For these figures, which represent structures of rotation, the following methods are common: disk method, shell method, surface of revolution.

In that study, the para-mathematical characteristic of most architectural domes is unfolded, the majority forms of which are dominated with curvature such as circumference, parabola or ellipse. These processes, whether isolation or integration methods, may help us to ascertain the geometrical bona-fide of these bodies and aid design and material prediction. And finally, Numerical integration methods are useful in situations where there is no equations in math.

These results further confirm that the definite integral is an apt mathematical basis for these expressions of curvature as architectural forms. These methods help architects and engineers to enhance design for structural analysis, material optimization, or even architecture-oriented applications. More contemporary building design may allow for further connections between mathematics and architecture through computational design and numerical modeling.

REFERENCES

[1] G. B. Thomas and R. L. Finney, *Calculus and Analytic Geometry*, 9th ed. Boston, MA, USA: Addison-Wesley, 1996. Available: https://books.google.com/books/about/Calculus_and_Analytic_Geometry.html?id=R8dvOfA_3XAC

[2] J. Stewart, *Calculus: Early Transcendentals*, 8th ed. Boston, MA, USA: Cengage Learning, 2016. Available: https://books.google.com/books/about/Calculus.html?id=EI4nAQAAIAAJ

[3] J. Stewart, *Calculus: Early Transcendentals*, official textbook resources. Available: https://www.stewartcalculus.com

[4] E. W. Weisstein, "Surface of Revolution," *MathWorld—Wolfram Research*. Available: https://mathworld.wolfram.com/SurfaceofRevolution.html

[5] K. Kolarevic, *Architecture in the Digital Age: Design and Manufacturing*. New York, NY, USA: Spon Press, 2003. Available: https://books.google.com/books/about/Architecture_in_the_Digital_Age.html?id=vxB0QgAACAAJ

[6] A. Pottmann, H. Asperl, M. Hofer, and A. Kilian, *Architectural Geometry*. Bentley Institute Press, 2007. Available: https://books.google.com/books/about/Architectural_Geometry.html?id=0M4kAQAAIAAJ

[7] S. Timoshenko and J. M. Gere, *Theory of Elastic Stability*. New York, NY, USA: McGraw-Hill, 2009. Available: https://books.google.com/books/about/Theory_of_Elastic_Stability.html?id=qP1RAAAAMAAJ

[8] C. J. K. Williams, "The Mathematics of Shell Structures," *International Journal of Space Structures*, vol. 19, no. 3, pp. 123-134, 2004. Available: https://journals.sagepub.com/home/iss

[9] M. Fox and M. Kemp, *Interactive Architecture*. New York, NY, USA: Princeton Architectural Press, 2009. Available: https://books.google.com/books/about/Interactive_Architecture.html?id=2o8bAQAAIAAJ

[10] J. Chilton, *The Engineer's Contribution to Contemporary Architecture*. London, UK: Thomas Telford Publishing, 2000. Available: https://books.google.com/books/about/The_Engineer_s_Contribution_to_Contempor.html?id=UjBMAAAACAAJ

[11] S. J. Gross, L. Valdevit, and A. Mohraz, "Shaping stress: How curvature governs the mechanics of film–substrate systems undergoing volumetric expansion," *npj Metamaterials*, vol. 2, no. 1, p. 9, 2026.

[12] L. Bouhala, S. Klein, S. Ozyigit, and A. Laachachi, "Process–structure–performance prediction of nanocomposite overwrapped pressure vessels: Manufacturing-driven design," *Composites Part C: Open Access*, vol. 19, p. 100711, 2026.

[13] R. Tonndorf, C. Elschner, A. Osterberg, V. Mozer, E. A. W. de Menezes, L. Römer, and M. Stommel, "Beyond decoration: Free-standing lace embroidery for 3D shaped surgical mesh implants," *Scientific Reports*, 2026.

[14] N. K. Solaisamy, P. W. David, P. Murugesan, S. Kavitha, and G. Sakthivel, “Experimental investigation of flexible solar panel performance on architecturally adaptive surfaces for optimal photovoltaic utilization,” *Solar Energy*, vol. 308, p. 114405, 2026.
[15] B. Q. D. Dammag, D. Jian, A. Q. Dammag, S. Almutery, A. Habibullah, and A. Baik, “A geospatially enabled HBIM–GIS framework for sustainable documentation and conservation of heritage buildings,” *Buildings*, 2026.