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Article

Mathematical Model of Acid Treatment of a Terrigenous Reservoir in an Oil Field

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Abstract: The article deals with the mathematical modeling of acid treatment processes of the near-wellbore zone of terrigenous reservoirs of oil fields. The function of chemical stimulation methods is analyzed under conditions of low permeability by which the oil recovery efficiency is still not enough. The interaction mechanisms between illite minerals and HF, reaction products and their effects on the rock parameters in sandstone reservoirs have been examined in detail. The general reaction–diffusion equations for the acid consumption and the mass transfer are described and combined into a system of equations governing the non-stationary filtration processes of oil and aqueous phases. The model additionally accounts for changes in porosity and permeability, build up of reaction products and gel formation that may restrict flow characteristics. In parallel, the methodological aspects of the model have implications. The foremost limitation arises from the omission of the gas phase and transitioning to a one-dimensional problem, which may limit the ability to make accurate quantitative predictions. However, the model serves as a conceptual framework for acid treatments to help understand the coupled physicochemical processes taking place in the near-wellbore zone during treatment.

Keywords: Breakeven, Hydraulic Fracturing, Reactant Fading, Upscaling, Well Stimulation, Capillary Pressure, Hydrocyanic Acid, Carbonate Reservoirs, Oil Production Rate, Silica Sand, Acid Reaction, Gas Production Rate, Mathematical Modeling

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1. Introduction

A significant share of the world's oil and gas reserves belongs to the category of hard-to-recover resources, since reservoirs in such fields possess low filtration–capacity properties (FCP) [1][2][3][4]. Under natural drive conditions and when applying secondary recovery methods—which themselves are insufficiently effective—the oil production rate and recovery factor remain low, while the recovery period is long [5]. Under these conditions, the development of various enhanced oil recovery methods yields positive results. The intensification of oil production is carried out through the bottomhole zone. Today, the most widely used method of recovery on the bottomhole zone is the chemical method. Chemical methods are applied in cases where the rocks of the bottomhole zone can be dissolved in various chemical reagents (for example, limestone in hydrochloric acid, sandstone or clay in mud acid, and others) [6][7].

The main mass of the terrigenous reservoir rock consists of silicate substances (quartz) and kaolins. These substances interact with hydrofluoric acid (HF), also known as fluoric acid.

Interaction with quartz occurs according to the following reaction:

$$SiO_2 + 4HF = 2H_2O + SiF_{4.}. \tag{1}$$

The resulting silicon fluoride $SiF_4.$ further reacts with water.

$$3SiF_4 + 4H_2O = Si(OH)_4 + 2H_2SiF_6. \tag{2}$$

Fluorosilicic acid H_2SiF_6 remains in the solution, while silicon hydroxide $Si(OH)_4$, as the acidity of the solution decreases, may form a gelatinous gel that can clog the pores of the reservoir.

Considering the practically instantaneous reaction of silicon fluoride with water, by combining equation (1) with equation (2), we obtain:

$$SiO_2 + 4HF = \frac{2}{3}H_2O + \frac{1}{3}Si(OH)_4 + \frac{2}{3}H_2SiF_6. \tag{3}$$

2. Methodology

Mathematical model

Considering the non-stationarity of the processes and their hydrodynamics, we assume that during filtration in an oil–water-saturated porous medium, the aqueous phase – which consists of acidic, saline, and water components—as well as the oil phase, participate in the process. The porous medium and the free particles formed as a result of its destruction dissolve in the acid and do not take part in filtration. The gas generated is completely dissolved in water and, due to its low concentration, is not taken into account in the model.

The system of equations for acid treatment of the bottomhole zones of oil reservoirs with terrigenous collectors can be presented as follows.

Equation for the mass conservation of the acidic component injected into the reservoir:

$$\frac{\partial}{\partial t}\left(m\rho_A c_A S_W\right) + \nabla \cdot \left(\rho_A c_A V_W\right) = -J_A + \nabla \cdot \left(\rho_A D m \nabla c_A S_W\right), \tag{4}$$

where $J_A = \begin{cases} 0 & , s_o > 0 \\ M_A aR & , s_o = 0 \end{cases}$ is the mass of acid consumed per unit time per unit volume, $R_A = E_f\left(\frac{c_A \rho_A}{M_A}\right)$ is the chemical reaction rate, $E_f = E_f^0 \exp\left(-\frac{\Delta E}{RT}\right)$ is the reaction rate constant, determined by the Arrhenius equation (ΔE is the activation energy, *R* is the gas constant), V_W is the filtration velocity of the aqueous phase, *m* is the porosity, ρ_A is the true density of the acid, c_A is the mass concentration of the acid, S_W is the water saturation of the pore space, D – is the molecular diffusion coefficient, t is time, $a = s/v$ is the specific reaction surface area, *s* is the reaction surface area, *v* is the reservoir volume, M_A is the molecular weight of the acid.

Equation for the mass conservation of silicon hydroxide formed as a result of the chemical reaction and injected into the reservoir.

$$\frac{\partial}{\partial t}\left(m\rho_S c_S S_W\right) + \nabla \cdot \left(\rho_S c_S V_W\right) = J_S, \tag{5}$$

where $J_S = \gamma_S J_A$ is the mass of silicon hydroxide generated by the reaction per unit time per unit volume, γ_S is the mass of silicon hydroxide produced per unit of silicon fluoride in the chemical reaction, c_S is the mass concentration of silicon hydroxide, ρ_S is the Density of silicon hydroxide.

Equation for the mass conservation of fluorosilicic acid formed as a result of the chemical reaction

$$\frac{\partial}{\partial t}\left(m\rho_{HS}c_{HS}S_W\right)+\nabla\cdot\left(\rho_{HS}c_{HS}V_W\right)=J_{HS}, \tag{6}$$

where $J_{HS} = \gamma_{HS} J_A$ is the mass of fluorosilicic acid produced by the reaction per unit time per unit volume, γ_{HS} is the the amount of hydrofluorosilicic acid mass transferred into water during a chemical reaction per unit mass of silicon tetrafluoride, c_{HS} is the mass concentration of hydrofluorosilicic acid, ρ_{HS} is the density of hydrofluorosilicic acid.

Mass conservation equation for the aqueous component formed as a result of the chemical reaction

$$\frac{\partial}{\partial t}\left(m\rho_W^0 c_W S_W\right)+\nabla\cdot\left(\rho_W^0 c_W V_W\right)=J_W^0, \tag{7}$$

where $J_W^0 = \gamma_W J_A$ is the mass of water produced by the reaction per unit time per unit volume, γ_W is the amount of mass transferred into water during the chemical reaction per unit mass of acid, c_W is the mass concentration of water, ρ_W^0 is the density of water.

Mass conservation equation of the aqueous phase

$$\frac{\partial}{\partial t}\left(m\rho_W S_W\right)+\Delta\cdot\left(\rho_W V_W\right)=J_W, \tag{8}$$

Where $J_W = -J_A + J_S + J_{SH} + J_W^0$, $\rho_W = c_A\rho_A + c_S\rho_S + c_{HS}\rho_{HS} + c_W\rho_W^0$

Mass conservation equation of the oil phase

$$\frac{\partial}{\partial t}\left(m\rho_o S_o\right)+\Delta\cdot\left(\rho_o V_0\right)=0, \tag{9}$$

where S_o is oil saturation, ρ_o is density of oil.

Porosity change equation

$$\frac{\partial}{\partial t}\left((1-m)\rho\right)=-J, \tag{10}$$

where, , $J = \gamma_M J_A$ is the mass of rock dissolved per unit time per unit volume, γ_M is the amount of rock dissolved during the chemical reaction per unit mass of acid, ρ_M *is* true density of the rock.

Filtration velocity equations of the water and oil phases

$$V_W = -\frac{Kk_W}{\mu_W}\nabla p, \quad V_o = -\frac{Kk_o}{\mu_o}\nabla p, \tag{11}$$

where *K*, k_W, k_o are absolute and relative phase permeabilities of water and oil $\mu_W = c_A\mu_A + c_S\mu_S + c_{HS}\mu_{HS} + c_W\mu_W^0$, μ_A, μ_S, μ_{HS}, μ_W^0 are viscosities of acid, silicon tetrafluoride, hydrofluorosilicic acid, and water, respectively, μ_o is viscosity of the oil phase, *p* is pressure.

Absolute permeability is expressed as follows:

$$K = K_0\left(\frac{m}{m_0}\right)^n, \tag{12}$$

changes in the specific surface area of the reaction

$$a = a_0\frac{(1-m)}{(1-m_0)}, \tag{13}$$

where a_0 - initial specific surface area.

Adding the obvious equalities

$$S_W + S_0 = 1,\ c_A + c_S + c_{HS} + c_W = 1 \tag{14}$$

equations of state:

$$\rho_\alpha = \rho_\alpha(p), \quad \alpha = A, S, HS, W; \tag{15}$$

dependencies for viscosities

$$\mu_\alpha = \mu_\alpha(p), \quad \alpha = A, S, HS, W; \tag{16}$$

and relative phase permeabilities

$$k_o = \begin{cases} \dfrac{(1-S_W)}{(1-S_{WE})} & S_W > S_{WE} \\ 1 & S_W \le S_{WE} \end{cases}, \tag{17}$$

$$k_W = \begin{cases} \dfrac{(S_1-S_{WE})}{(1-S_{WE})} & S_W > S_{WE} \\ 0 & S_W \le S_{WE} \end{cases}, \tag{18}$$

initial conditions

$$p(x,0) = p^0,\ m(x,0) = m^0,\ S_W(x,0) = S_W^0,\ S_o(x,0) = S_o^0,$$
$$c_\alpha(x,0) = c_\alpha^0,\ \alpha = A, S, HS, W; \tag{19}$$

boundary conditions

$$p(0,t) = p^c(t),\ \ S_w(0,t) = 1,\ S_O(0,t) = 0,\ c_\alpha(0,t) = c_\alpha^c(t),$$

$$\alpha = A, S, HS, W; \left| p(L,t) = p^0 , \left. \frac{\partial c_A}{\partial x} \right|_{x=L} = 0 \right. \tag{20}$$

3. Results

Equipping the above set of equations with initial and boundary conditions yields a closed system of equations (14)–(20) that describes the succession of physicochemical processes in the bottomhole zone of a terrigenous reservoir involved in acid treatment, with regard to chemical kinetics. The developed model demonstrates through analysis that acid concentration along the path of filtration decreases because of its consumption in reactions with the rock matrix. This results in a spatially heterogeneous distribution of reactive capacity in the reservoir. Secondary reaction products include fluor silicic acid and silicon hydroxide resulting from the interaction of hydrofluoric acid and silicate minerals [8]. Under conditions of falling acidity, the latter can accumulate as a gel-like phase in the pore space [9].

The model also suggests that increased porosity is caused by dissolved rock matrix. This process, however, is not entirely monotonic, as the formation and deposition of reaction products can mitigate the dissolution effects [10].

Variation of permeability is controlled by the effects of pore radius increase and blocking. This can lead to both the enhancement and reduction of permeability in different zones of the reservoir, depending on reaction intensity and transport conditions [11].

Hence, such a system of equations allows to describe the competing processes of enlargement of the pore space and clogging, which govern the overall efficiency of acid treatment in terrigenous reservoirs [12].

Since the bottomhole zone is symmetrical with respect to the well, for a qualitative assessment of the processes occurring within it, one can transition from a three-dimensional model to a one-dimensional model [13].

4. Discussion

The developed mathematical model represents an integral account of physicochemical processes in the near-wellbore zone of a terrigenous reservoir with chemical-acidization treatment. In comparison to the simplified approaches, it considers the filtration, the chemical reactions, and the structural changes of the porous medium together.

However, some of the assumptions are less than ideal and deserve careful scrutiny before drawing any conclusions from the model. Specifically, neglecting all of the gas-phase (which produces during the chemical reactions) due to assuming completely dissolved in the aqueous phase. Although this assumption simplifies a lot of the formulation this is not always valid, particularly at higher temperatures or in very reactive environments [14].

The second limitation is due to the model being reduced from 3D to 1D. Although this hypothesis is physically sound owing to symmetry and enables qualitative analysis, it limits the extent of heterogeneous effects to be examined, a common feature in nature beds that have terrigenous sediments.

Moreover, the model assumes that any solid particles produced during the dissolution of rock are not involved in filtration. In reality, the subsequent migration and deposition of suspended particles has a strong potential to alter pore structure and permeability evolution [15].

Perhaps most interesting, the model shows the conflicting role of acid treatment. Increased dissolution of the rock matrix would increase porosity and also increase possible permeability in one aspect. Conversely the precipitated secondary products, particularly silicon hydroxide, could plugging their pores. This means acid treatment is not something that we can evaluate unambiguously and that the effectiveness of this kind of treatment will have to be viewed in light of competing processes that are trying to run.

This makes the model usable, in a qualitative sense, for analysing acid treatment processes, but as we found due to the many simplifying assumptions that have been made to arrive at the model, its use for quantitative prediction is limited and needs further refinement and possibly the relaxation of some of the simplifying assumptions.

5. Conclusion

The mathematical model developed describes the coupled physicochemical processes that occur in the zone of the influence of acid treatment in the near-wellbore of a terrigenous reservoir to represent filtration and geochemical reactions as well as the changes in reservoir properties. This assumption leads to a closed system of equations for the acid — rock interaction and the reaction products.

Simultaneously, the model shows that the effects of acid treatment are not one way. In addition to the rock dissolution leading to increased porosity, secondary processes connected to the development of reaction products may affect the pore structure and flow properties.

These assumptions include neglecting the gas phase and simplifying to a quasi-one-dimensional formulation, both of which make the model more tractable but are useful for qualitatively understanding the processes at play. In the process of deriving this model the authors made several simplifications; thus, the model may potentially fail to predict detail quantitative accurately.

Therefore, the approach seems to be a fairly theoretical basis for application of acid treatment in terrigenous reservoirs, while the model needs further refinement to provide a more realistic description of actual reservoir conditions.

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