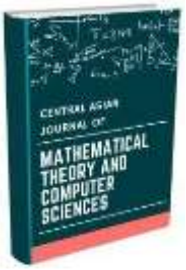

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Article

Efficiency of Bayesian Estimator of Shrinkage by Weighted Loss and Exponential Linear functions for Frecht`s Distribution by Using the Simulation Method

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Abstract: This work deals with study of Bayesian shrinkage for parameters of the Frecht`s distribution by following two kinds of loss functions. These are weighted loss function and linear exponential loss function. The simulation method is depended as main manner using the Stata17 program to evaluate the efficiency of the estimators and to find the best Bayesian shrinkage estimator. Bayesian estimators have been derived and then both estimators were calculated by using standardized performance measures to make multi-condition simulation data including six different paths for feasible simulation. Each path checks a different aspect for an effect of sample size and calculates the estimators of the weighted and linear exponential loss functions. After that, a comparison between them is made aiming the arrival to a best estimator.

Keywords: Frecht`s Distribution, Bayes' Theorem, Probability Density Function, Weighed Loss Function, Linear Exponential Loss Function, Accumulative Density Function, shrinkage estimator.

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1. Introduction

Bayesian estimation is considered to be best efficient and flexible statistical manner in processing of estimation problems such as improving accuracy of estimators following combining the previous data with practical data to reach estimators that be realistic for the unknown data [1]. In addition to enhancing of estimation efficiency especially the data that be small sample or uncertain. The loss function is one of the basic element in Bayes` theory because it is used to know amount of loss resulting from testing a specific estimation [2].

In this research, Bayesian concept of shrinkage is applied on Frecht`s distribution because this distribution has an importance in massive data and applications in different fields. It is considered that estimation of parameters of this distribution is a large challenge according to updating its experiments [3]. There are many studies had been referred to Bayes`s estimation for different statistical distributions, but the application of Bayesian estimator of shrinkage is still limited, especially that subject to two types of loss functions: the weighted loss function and the linear exponential loss function [4]. Beside that, the comparison of performance these estimators undergoing different conditions did not study deeply [5]. The problem is that choice of an suitable loss function will impact mainly in acting of the Bayesian estimator, so it is necessary to determine efficiency of Bayesian estimator of shrinkage depending on a simulation method reaching to more reliable and accurate estimation [6].

2. Materials and Methods

This paper researches an application of concept of Bayesian estimator of shrinkage on Frecht`s distribution subject to different two loss functions which are weighted and linear exponential with estimation of parameters of the distribution depending on using Bayes' method. Furthermore, comparison of efficiency of both estimators under these functions and comparison the results obtaining from traditional estimators. The Simulation manner is used to evaluate acting of estimators and analyzing impact of kind of loss function on accuracy of results and estimation quality.

This distribution is named relative to the French mathematician Maurice René Fréchet (1878-1973), who developed in the 1920s as a maximum value distribution (also known as the second-order maximum value distribution). Frecht`s distribution is considered one of the probability distributions for life-time models. This distribution described and discussed by Kotz and Nadarajah to apply for natural disasters, rainfalls, horse racing, supermarket queues, life tests . Also it is used in modeling failure rates, which are commonly used in biological studies, light signal analysis and error model building.

Probability Density Function

The researcher (Drapella) presented and researcher (Mundhol karad kollia) proposed a name for Inverse Weibull distribution on Frecht`s distribution.

If the random variable(t) has Weibull distribution, then the variable x = 1/t

The Frecht`s distribution and the probability density function are represented as follows:

$$f(x.\alpha.\lambda) = \alpha\lambda^{\alpha}x^{-(\alpha+1)}e^{-\left(\frac{\lambda}{x}\right)^{\alpha}} \quad \dots\dots\dots\dots\dots (1)$$

Where :

$x > 0$,and the shape parameter is $\alpha > 0$, and the scale parameter $\lambda > 0$.

The figure (1) shows the probability density function of the Frecht`s distribution for different values of α and λ:

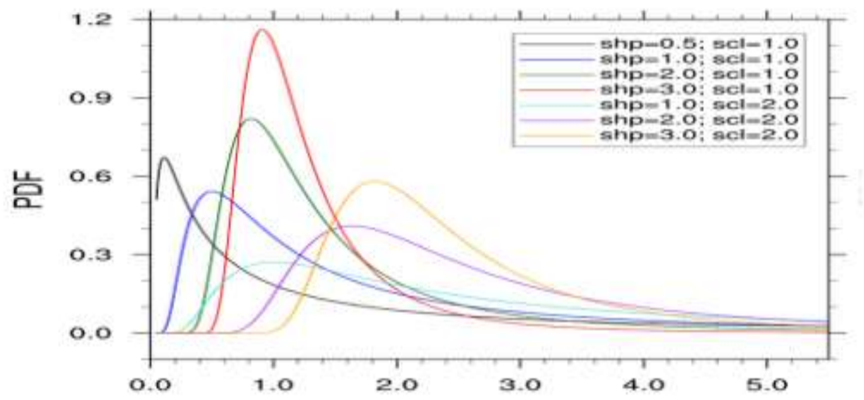

Figure 1. Density Function Curve probability to distribute Frecht`s for different values of α and λ

Accumulative density function[1]

If (x) is a random variable distributes in the Frecht`s distribution. x~$(\alpha.\lambda)$,than its accumulative function F(x) will be as follows:

$$F(x.\alpha.\lambda) = P(X \le x) = \int_{-\infty}^{x} f(u)du \quad \dots\dots\dots\dots\dots\dots (2)$$

$$F(x.\alpha.\lambda) = \int_{0}^{x} \alpha\lambda^{\alpha}u^{-(\alpha+1)}e^{-\left(\frac{\lambda}{u}\right)^{\alpha}} du$$

$$F(x.\alpha.\lambda) = e^{-\left(\frac{\lambda}{x}\right)^{\alpha}} \quad ; \quad x \ge 0 \quad \dots\dots\dots\dots\dots\dots (3)$$

and the figure (2) shows the a cumulative probability distribution function of the Frecht`s distribution for different values of α and λ:

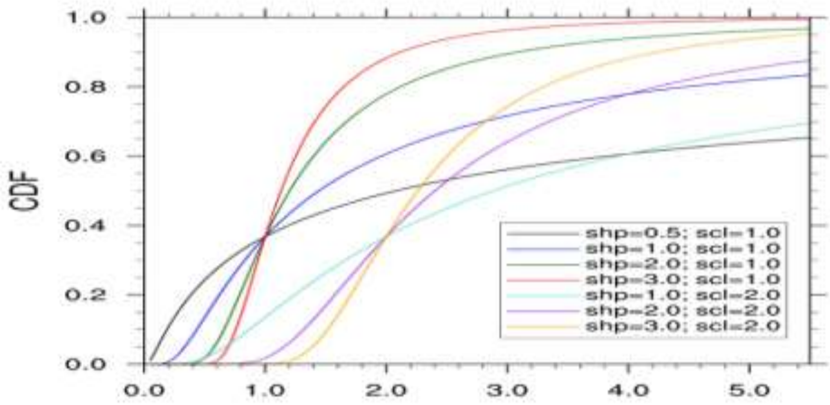

Figure 2. Distribution Function probabilistic accumulative Frecht`s distribution for different values of α and λ

3. Results and Discussion

Bayes' theory

Bayes' theory is considered a foundation of Bayesian inference statistics. This theory supposes that the unknown parameters be as random variables. And there are previous data formulated as a probability distribution known as the probability density function if it is possible can be identified from the theory that governs the phenomenon or from previous data and experiments [7]. Bayes' theory also depends on the current information of the sample, which is represented by the maximum probability function of the observations. The following probability distribution by combining the initial probability density function of the parameters with the maximum probability function of the current observations [8]. For explain Bayes' method of estimation in general terms, after obtaining the next probability distribution, what is known as the loss function is determined [9]. By this function, it is possible to measure the loss resulting from making decisions depending on the ($\hat{\lambda}$), but the suitable decision depends on (λ). i.e. there is a difference between the value of the parameter and its estimation. The basic elements of Bayes' method are:

1. Knowing the previous probability $\pi(\lambda)$ (before the inspection), which is obtained from previous or applied studies.
2. Knowing the probability function $f(\lambda)$ of the data distribution, from which we obtain the maximum likelihood function of the observations.
3. Obtaining the subsequent probability $h(x_1.x_2.\dots.x_n)$ (after the inspection) according to Bayes' theory. We obtain the subsequent distribution using Bayes' inverse formula as follows:

$$h(\lambda|x_1.x_2.\dots.x_n) = \frac{\pi(\lambda)\prod_{i=1}^{n} f(x_1.x_2\dots.x_n|\lambda)}{\int_{\forall\lambda}\pi(\lambda)\prod_{i=1}^{n} f(x_1.x_2\dots.x_n|\lambda)} \quad \dots\dots\dots\dots\dots (4)$$

Where:

$\pi(\lambda)$: Initial distribution of the parameter (λ).

$f(x_1.x_2\dots.x_n|\lambda)$: The maximum likelihood function of the sample observations.

$h(\lambda|x_1.x_2.\dots.x_n)$: Subsequent distribution.

Estimated Shrinkage

Shrinkage theory is based on the assumption that the unknown parameter which we need to estimate is a random variable for a certain distribution. This theory depends on the shrinkage parameter θ and the acceptance domain K, where the shrinkage parameter means the degree of confidence of the researcher with initial information. Each researcher can determine a formula for choosing a value of θ according to rules that he/she believes are sufficient, because of the lack of a unified formula for choosing the value of θ [10].

Thompson suggested that formula of estimator for shrinkage is follows:

$$\hat{\theta}_{sh} = k\hat{\theta} + (1-k)\theta_0 \qquad . \ 0 \leq K \leq 1 \qquad \dots\dots\dots\dots\dots (5)$$

Where:

θ_0 :The initial information for the parameter θ.

$\hat{\theta}$: An unbiased initial estimator for the parameter θ.

loss function

The loss function is defined as a measure of the amount of loss resulting from a making decision that depends on the estimator ($\hat{\theta}_{Bayes}$). Bayesian estimators are different depending on the type of loss function where the availability of them is essential, unlike other methods in estimating of parameter or vector of parameters. When the Bayesian estimator is obtained, the expected subsequent loss is minimized [11]. This estimator is mainly related to the hazard function making at minimizing value which is obtained by evaluating the expectation for the loss

function that is denoted by the L(θ̂,θ) because of accuracy of the Bayesian method in obtaining the estimated parameter.

If θ is the distribution parameter which we need to estimate for, then L(θ̂,θ) will be loss function satisfying the following conditions:

1. L (θ̂,θ) ≥ 0 ; ∀ θ . θ̂
2. L(θ̂,θ) = 0 ; ∀ θ̂ = θ

Bayesian Shrinkage estimator subject to a weighted loss function

$$L_3\left(b.\ \hat{b}_{s3}\right) = w\frac{\sum_{i=1}^{n}\left(x_i - \hat{b}_{s3}\right)^2}{b^2} + (1-w)\left(\frac{\hat{b}_{s3}}{b} - 1\right)^2 \qquad \dots\dots\dots (6)$$

Where $0 \leq w \leq 1$

$$\rho_3\left(b.\hat{b}_{s3}\right) = \frac{w}{n}\sum_{i=1}^{n}\left(x_i - \hat{b}_{s3}\right)^2 Eb\left(\frac{1}{b^2}\right) + (1-w)Eb\left[\left(\frac{\hat{b}_{s3}}{b} - 1\right)^2\right] \qquad \dots\dots\dots\dots (7)$$

Taking the derivative basing on ($\hat{b}_{s3}$) and making it equal to zero, then:

$$\hat{b}_{s3} = w\bar{x} + (1-w)\frac{E[b^{-1}]}{E[b^{-2}]} \qquad \dots\dots\dots\dots (8)$$

Where $\bar{x}$: is arithmetic mean, which is equal to $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$

and $b^{-1} \sim inversgamma$

Where:

$$E\left(\frac{1}{b}\right) = E(b^{-1}) = \frac{t}{(n-1)}$$

and

$$E\left(\frac{1}{b^2}\right) = E(b^{-2}) = \frac{t^2}{(n-1)(n-2)}$$

Therefore, he estimated value ($\hat{b}_{s3}$) will be

$$\hat{b}_{s3} = w\bar{x} + (1-w)\frac{n-2}{\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha}} \qquad \dots\dots\dots\dots\dots (9)$$

It represents the total Bayesian shrinkage by a weighted loss function

Where:

$$t = \sum_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha}$$

and by putting (w=0) then:

$$\hat{b}_{s3} = \frac{n-2}{\sum_{i=1}^{n} \left(\frac{1}{x_i}\right)^{\alpha}}$$

and putting (w=1), then: $\hat{b}_{s3} = \bar{x}$

The subsequent risk function is:

$$k = \frac{w\bar{x}-b_0}{\left[w\bar{x}+\frac{(1-w)(n-2)}{t}\right]-b_0} + (1-w)\frac{(n-2)}{t\left[\left(w\bar{x}+\frac{(1-w)(n-2)}{t}\right)-b_0\right]} \qquad \dots\dots\dots\dots (10)$$

The shrinkage theory is defined as follows:

$$\hat{b}_{sh3} = k\left(\hat{b}_{s3} - b_0\right) + b_0$$

By substituting equation (11) into (12), we obtain the Bayesian shrinkage estimator for the Frecht`s distribution subject to a weighted loss function:

$$\hat{b}_{sh3} = \left[\frac{w\bar{x}-b_0}{\left[w\bar{x}+\frac{(1-w)(n-2)}{t}\right]-b_0} + (1-w)\frac{(n-2)}{t\left[\left(w\bar{x}+\frac{(1-w)(n-2)}{t}\right)-b_0\right]}\right] * \left[\left(w\bar{x} + (1-w)\frac{n-2}{\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha}}\right) - b_0\right] + b_0 \qquad \dots\dots\dots\dots (11)$$

<u>Bayesian Shrinkage estimator subject to linear exponential loss function (LINEX)</u> [8],[4]

$$\rho\left(\hat{b}.\hat{b}_{s2}\right) = E\left[e^{a\left(\frac{\hat{b}_{s2}}{b}-1\right)} - \alpha\left(\frac{\hat{b}_{s2}}{b} - 1\right) - 1\right] \qquad \dots\dots\dots\dots (12)$$

ρ : is the correlation function between the parameter and the Bayesian estimator in the assumed loss function.
$\hat{b}_{s2}$: is the Bayesian estimator in the second loss function (LINEX).
By simplifying, we obtain a Bayesian estimator subject to a loss function LINEX :

$$\hat{b}_{s2} = \frac{1}{\alpha}\left[t - \left(nt^{n-1}e^{-a}\right)^{\frac{1}{n+1}}\right] \qquad \dots\dots\dots\dots (13)$$

The shrunk Bayesian estimator and dependent on the loss function is :

$$\hat{b}_{sh2} = \left[\frac{\left(\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha} - ab_0\right) - \left[n(n-1)\left(\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha}\right)^{n-1} e^{-a}\right]^{\left(\frac{1}{n+1}\right)}}{a\left(\left[\left(\frac{1}{\alpha}\right)\left[\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha} - \left(n\left[\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha}\right]^{n-1} e^{-a}\right)^{\frac{1}{n+1}}\right]\right] - b_0\right)}\right] * \left[\left(\frac{1}{\alpha}\left[\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha} - \left(n\left[\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha}\right]^{n-1} e^{-\alpha}\right)^{\frac{1}{n+1}}\right]\right) - b_0\right] + b_0 \qquad \dots\dots\dots\dots (14)$$

<u>Practical part</u>

The practical part of the research will rely on generating multi-group data and then calculating both estimators for each group using standardized performance measures to ensure a logical and statistically valid comparison. Thus, the primary objective of the practical part will be to compare the performance of Bayesian estimators of shrinkage subject to loss functions, according to the following steps [12].

First: Generating of data , multi-condition simulation.

We will develop six different scenarios for a scientifically feasible simulation without complexity. Each scenario will test a different aspect of the sample size effect, which will be tested in scenarios one, two, and five. The sample size will be small in scenario one (30 observations), medium in scenario two (50 observations), and large in scenario three (100 observations). The effect of the distribution parameters will be tested in scenarios one, three, four, and six, with the effect will be basic in scenario one, heavy-tailed in scenario three, light-tailed in scenario four, and extreme in scenario six [13]. The efficiency design is dependent to choose these scenarios. Stata 17 is used to make a simulation which is multi-condition to generate data which is Frecht`s distributed and application of shrunk Bayesian estimators subject to weighted loss and linear exponential functions. Table (1) provides an explanation of these scenarios according to sample size and the basic description of the scenario.

Table 1. Generation of multi-condition simulation data

DESCRIPTION	SAMPLE SIZE	λ	α	THE SCENARIO
basic condition	30	1	2	1
large sample	100	1	2	2
Different parameters	30	1.5	1.5	3
Different parameters	30	0.8	2.5	4
average sample	50	1	2	5
extreme parameters	30	1.2	3	6

Secondly: Calculating estimators and other measures

1. Estimator of the weighted function from equation (11) which was calculated by Stata 17 is

$$\hat{b}_{sh3} = k(\hat{b}_{s3} - b_0) + b_0$$

Initially, the calculation of **b_hat_s3**, based on equation (9)
Local b_hat_s3 = w * mean_x + (1-w) * (n_obs -2)/ t
The shrinkage factor then will be calculated from equation (10)
Local k_num1 = w * mean_x - b0
Local k_denom1 = (= w * mean_x +
Local k_num2 = (1-w) * (n_obs -2)
Local k_denom2 = t * ((w * mean_x + (1-w) * (n_obs -2)/t) - b_0)
Local k = k_num1 / k_denom1 + k_num2 / k_denom2
Estimating the final estimator of the weighted function from equation (11)
Local b_hat_sh3 = k * b_hat_s3_b_0) +b_0

2. Calculating the estimator of a linear exponential function is as follows:
Function estimator LINEX
Local part1 = (1/ alpha) * (t – n_obs * t^(n_obs-1) * exp a))^(1/(n_obs+1)))
Local k_linex_num1 = (t - a*b0) – (n_obs*(n_obs -1)*(t)^(n_obs – 1)*exp(-a))^(1/(n_obs+1))
Local k_linex_denom1 = a * (part1 – b_0)
Local k_linex = k_linex_num1/ Local k_linex_denom1
Local b_hat_sh2 = k_linex * (part1 -b0) + b_0
'

3. Calculating of accuracy measures , as follows:
Local mse_weighted = (b_hat_sh3 – lambda)^2
Local mse_linex = (b_hat_sh2 – lambda)^2
Local efficiency = mse_linex / mse_weighted
Local best_method = LINEX if mse_linex < mse_weighted
Replace Local best_method = Weighted if mse_weighted ≤ mse_linex

The results are shown as in table (2) below.

Table 2. Results of multi-condition simulation according to the program Stata 17

scenario	α	λ	n	b_hat_~3	3 b_hat_~2	mse_we~d	mse_li	> ~x	effici~y	best_m~d
1	2	1	30	1.15	0.97	0.048	0.0	> 38	0.79	LINEX
2	2	1	100	1.04	0.99	0.013	0.0	>9	0.69	LINEX
3	1.5	1.5	30	1.62	1.67	0.071	0.0	>72	1.01	Weighted
4	2.5	0.8	30	0.79	0.82	0.029	.00	>25	0.86	LINEX
5	2	1	50	1.08	1.02	0.023	0.0	>19	0.83	LINEX
6	3	1.2	30	1.25	1.28	0.038	0.0	>41	1.08	Weighted

Source: Prepared by the researchers based on simulation results using the program Stata 17

Third: Interpretation of the results according to the scenarios.

1. Scenario 1: The results have shown according to this scenario tell us shrunk Bayesian estimator by a linear exponential loss function provides more accuracy by a percentage 21% and Its estimation is closer to the real value than the weighted estimation [14].
2. Scenario 2 : The results have shown that shrunk Bayesian estimator by a linear exponential loss function provides higher efficiency by percentage31% which is very near to the real value and it is much better than the weighted estimation.
3. Scenario 3:The results has shown that with different parameters, the weighted estimator is better than shrunk Bayesian estimator by a linear exponential loss function with proportion1% which is a small difference.
4. Scenario 4:By this scenario, the results have shown that shrunk Bayesian estimator by a linear exponential loss function provides more accuracy with a percentage 14% and its estimation is closer to the real value compared to the weighted estimation.
5. Scenario 5:In the average sample which is 50, the shrunk Bayesian estimator by a linear exponential loss function supplies more accuracy by a percentage 17% of the weighted estimator.
6. Scenario 6: The weighted estimator with extreme parameters is superior to shrunk Bayesian estimator by a linear exponential loss function to provide more accuracy with a percentage 8%.

Fourth :Sample Impact

The results of the multilateral simulation appeared that sample size has an effect on the superiority of one variable on each other in matter of efficiency, as follows::

1. In the small samples (30 observations), the shrunk Bayesian estimator by linear exponential loss function shows lower efficiency.
2. In the average samples (50 observations), shrunk Bayesian estimator for a linear exponential loss function is more efficient.
3. The large samples (100 observations), have appeared better efficient for shrunk Bayesian estimator by a linear exponential loss function.

Thus, increasing of sample size will play an important role in increasing of efficiency for shrunk Bayesian estimator by a linear exponential loss function.

Fifth : Impact of distribution parameters

The results of the multi-party simulations have shown that the distribution parameters have an effect on which estimator is best, and as follows::

1. In standard parameters, the results of multi-party simulations showed that shrunk Bayesian estimator by linear exponential loss function will be superior to the weighted estimator.
2. In extreme parameters, the weighted estimator will be more superior.
3. In the different parameters, acting of both estimators is a bit similar.

According to the results of simulation, we can say that the linear exponential shrunk Bayesian estimator by the loss function is superior in four scenarios taken out six scenarios. Efficiency of this estimator is ranging between 68% and 69% explaining its superiority by percentage 14% to 31% if it is compared to the weighted function estimator. Its average efficiency is 79% within six scenarios which dependent in multi-condition simulation [15].

In large samples, linear exponential shrunk Bayesian estimator is characterized by a high performance to achieve efficiency 69% and showing very near estimations from the real values in many scenarios.

In extreme parameters and different values of parameters, the weighted function appeared its superiority in these conditions.

In many case, researchers suggest by adopting the shrunk Bayesian estimator by the linear exponential loss function, whereas in singular cases, the weighted function is recommended.

The following table (3) summarizes all the results above.

Table 3. Summary of conclusions drawn from the results of multi-condition simulation

Recommendation	Superiority ratio	The best	conditions	The scenario
Recommended	21%	Linear exponential	The sample is small with standard parameters.	1
Recommended	31%	Linear exponential	The sample is large with standard parameters.	2
Any both	1%	The weighed	Different parameters	3
Recommended	14%	Linear exponential	Different parameters	4
Recommended	17%	Linear exponential	The sample is average.	5
Weighted is recommended	8%	The weighed	The parameters are extreme	6

Source: From the work of researchers based on the results of multi-condition simulation

Sixth : The graphs

In context of statistical researches and for supporting statistical analysis, the figure (3) is made to recognize the visual image to perform the estimators as shown below:

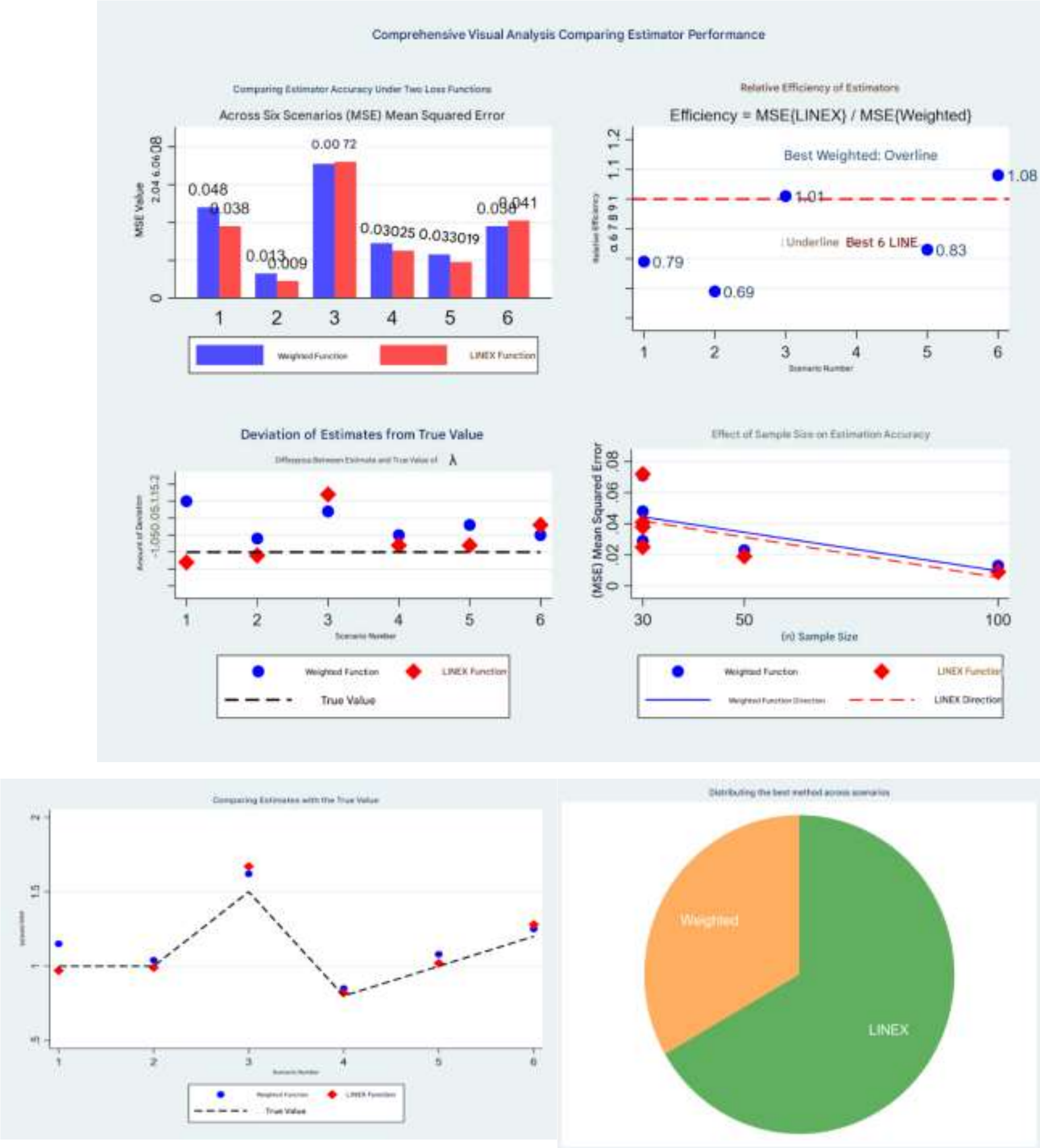

Figure 3. Comprehensive visual analysis to compare the performance of the estimators

4. Conclusion

The following elements are appeared by the research:

1. The linear exponential loss function is the more best of shrunk Bayesian estimator by weighted loss function and exponential loss function for Frecht`s distribution.
2. The shrunk Bayesian estimator by the exponential loss function provides higher accuracy and it is closer to the truth.

3. The shrunk Bayesian estimator by the exponential loss function shows higher efficiency and it is close to the ideal situation.
4. When the sample size increases, the shrunk Bayesian estimator by the exponential loss function appears higher efficiency.
5. The estimator of a weighted function will be higher efficiency in case of the more extreme parameters.
6. Choosing of the appropriate loss function, whether it is linear exponential or weighted, is not an arbitrary decision. But this choice depends on the nature of data, sample size and true values of the unknown parameters.
7. The methodology of multi-condition simulation has proven to be successful for statistical test to evaluate and select the best estimators with multiple probabilities. This methodology has shown comprehensive view without any doubt to perform the estimators.

Recommendations

Through the statistical analysis and in the view of the preceding conclusions , many researchers recommend the following considerations.

1. In general, the shrunk Bayesian estimator by linear exponential loss function is suggested when we want to estimate the parameters of Frecht`s distribution.
2. It is necessary to use shrunk Bayesian estimator by linear exponential loss function when we want to estimate parameters of Frecht`s distribution in case of large samples and standard parameters because this usage will be very efficient and reliable.
3. In case of the extreme parameters, it is preferable to dependent on shrunk Bayesian estimator by the weighted loss function when we want to estimate parameters of Frecht`s distribution to satisfy more accurate results.
4. It is useful to enlarge size of samples as possible, to enhance accuracy and efficiency of estimating parameters of Frecht`s distribution.
5. The exploratory analysis for nature of data is crucial for selecting an appropriate loss function to estimate the parameters of Frecht`s distribution aiming to determine domain of possible values for parameters and nature of distribution.

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