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Article

Optimal Parameters of Dynamic Absorber for The Vibrations of The Beam With Moving Dynamic Absorber

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Abstract: Vibration control is of utmost importance in modern mechanical and structural engineering in which excessive oscillations can cause the failure of operations, noise, fatigue, or even the collapse of the structure. Dynamic vibration absorbers provide an effective, simple and energy-efficient solution for transferring vibrational energy of the primary system to the secondary subsystem, which reduces the amplitude of the main structure. Beams are highly common structures present in bridges, vehicles, robotics, and aircraft, which exhibit complex vibrations with dynamic or moving loads, such as that of a vehicle or a crane. Consequently, a moving DVA, which moves along the beam optimizing the suppression method is required, and with parameters such as mass ratio, damping, stiffness, and position liquidating the performance of the device on such system. Although DVAs are relatively well-developed, on beams with hysteresis-type elastic dissipative characteristics, under moving loads, improper tuning of the parameters can result in reduced functionality or even increase the vibrations of the structure at certain frequencies. Consequently, this study aims at finding such optimal parameters for moving DVA on a beam and absorber with hysteresis-type elastic dissipative characteristics using purely analytical methods. As a result, the optimization allowed to reduce the amplitude and energy transmission, the parameters for which were equal to 0.1253, 1.92 and 0.5 respectively. The results were obtained through analytical modeling, resonant analysis and the Den Hartog method, were at- and transmitted power were equal to zero, are used to find the transfer function equations and the invariant points of the system under kinematic and random excitations. Numerical analysis provided optimal mass, stiffness, and damping for the moving DVA on a clamped end steel beam. The amplitude-frequency characteristic results are highlighted in that transmission is limited in the vicinity of the resonant frequencies, and the curves are shifted to the right with absorber tuning when moving and the peaks are the lowest. The work is unique in its kind as it performs such optimization using the Ginzburg method and analytical methods in order to achieve optimal suppression rather than rely on the experiment only. Results of the study can be applied to optimize values of DVAs on hysteresis beam for vibration-sensitive structures like robotic arms and high-speed railways, in order to employ maximum suppressional capabilities of the device under dynamic excitations.

Keywords: beam, dynamic absorber, vibrations, transfer function, kinematic and random excitations.

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1. Introduction

Currently, clear and perfect performance of techniques provide current from problems is considered. In this regard in them appearance to be harmful vibrations quench requiring solution from problems is one [1].

Vibration control has become one of the most essential areas of modern mechanical and structural engineering, as excessive oscillations can lead to serious operational failures, noise, fatigue, or even catastrophic collapse of engineering systems. Among various vibration mitigation techniques, the use of dynamic vibration absorbers (DVAs) has proven to be one of the most effective and practical solutions due to their simplicity, energy efficiency, and adaptability to a wide range of mechanical structures. The fundamental principle of a DVA lies in transferring the vibrational energy from the primary system to a secondary subsystem (the absorber) designed to oscillate in such a way that it counteracts and reduces the amplitude of vibrations of the main structure [2].

Beams are one of the most common structural elements in mechanical and civil engineering applications they appear in bridges, vehicle components, robotic manipulators, and aircraft wings [3]. Because of their widespread use, the suppression of beam vibrations has been extensively studied. However, when the excitation is dynamic or the load is moving (as in cases of moving vehicles, cranes, or robotic systems), the vibration characteristics become more complex. In such conditions, a moving dynamic absorber presents a unique approach: instead of being fixed at one point, the absorber can travel along the beam, adjusting its position to achieve optimal vibration reduction [4]

The determination of optimal parameters such as the absorber’s mass ratio, damping coefficient, stiffness, and its position along the beam plays a crucial role in achieving effective vibration suppression. Improper parameter selection can not only reduce the performance of the absorber but may even amplify the vibration amplitude at certain frequencies. Therefore, developing analytical and numerical methods for optimizing these parameters has attracted significant attention from researchers in recent decades [5].

2. Materials and Methods

This study focuses on investigating the optimal parameters of a moving dynamic absorber for a beam subjected to vibrations. The analysis aims to derive conditions that minimize the vibration amplitude and energy transmission under various dynamic loading scenarios. Both theoretical modeling and computational simulations are utilized to determine the absorber’s ideal characteristics and trajectory along the beam. The results of this research can contribute to the design of intelligent vibration control systems in engineering applications such as flexible robotic arms, high-speed railways, and aerospace structures, where dynamic loading and moving masses are unavoidable.

Mechanics systems nonlinear characteristics into account received without, mathematical modeling, in which resonance analysis and to determine optimal parameters with optimization methods related many scientific research works take visited.

Determining the optimal parameters of a dynamic absorber is considered in mechanic a complete analysis of the system and the performance of each parameter It is necessary to assess the impact on the system. Mathematical modeling, resonance analysis and determining optimal parameters with optimization methods possible . Initially optimization method selectively we get, the optimal values of the parameters in determining Lagrange, Hamilton and Den Hartog from the methods use possible. Today on the day mechanic effects assessment in terms of mechanic systems vibrations in extinguishing being used dynamic optimal parameters of absorbers Den Hartog in the identification method other to methods relatively wide For this purpose, Den Hartog from the method used without transmission function absolute value for following relationships harvest we do :

$$\left[(1+(M_0-A_0-2)K_2)\frac{n_0^2}{p_i^2}+\eta_2(C_0+H_1)\right]\frac{\omega^4}{p_i^4}+[(((K_1-1)(A_0+2)\eta_2-$$
$$-(-1+(A_0+2)K_2)\eta_1)C_0-1+(A_0+2)K_2)-(K_1-1)(A_0+1)\times$$
$$\times(2K_2-1)\frac{n_0^2}{p_i^2}+H_1(\big((K_1-1)\eta_2-K_2\eta_1\big)(A_0+2)+\eta_1)]\frac{n_0^2}{p_i^2}\frac{\omega^2}{p_i^2}+$$
$$+(A_0+1)[\big(C_0\eta_2((K_1-1)^2+K_2^2+K_2)-(2K_2-1)(C_0\eta_1-1)(K_1-1)\big)+$$

$$+(\eta_2((K_1-1)^2+K_2^2+K_2)-\eta_1(2K_2-1)(K_1-1))H_1]\frac{n_0^4}{p_i^4}=0;$$

$$\left[(1+(M_0-A_0)K_2)\frac{n_0^2}{p_i^2}+\eta_2(C_0+H_1)\right]\frac{\omega^4}{p_i^4}+[(((K_1-1)(A_0+1)\eta_2+ \qquad (1)$$
$$+(1+A_0K_2)\eta_1)C_0-1-A_0K_2)+((K_1-1)(A_0+1)\times$$
$$\times(2K_2+1)\frac{n_0^2}{p_i^2}+H_1(((K_1-1)\eta_2+K_2\eta_1)A_0+2(K_1-1)\eta_2+\eta_1)]\frac{n_0^2}{p_i^2}\frac{\omega^2}{p_i^2}+$$
$$+(A_0+1)[(C_0+H_1)((K_1-1)^2-K_2^2-K_2)\eta_2+$$
$$+(C_0\eta_1-1+\eta_1H_1)(K_1-1)]\frac{n_0^4}{p_i^4}=0,$$

where

$$A_0=d_i\mu\mu_0u_{i0};\ M_0=\mu\mu_0u_{i0}^2;\ \mu=\frac{m}{\rho Al};\ \mu_0=\frac{l}{d_{2i}};\ d_i=\frac{d_{1i}}{d_{2i}};\ d_{1i}=\int_0^l u_i\,dx;$$

l is beam length; $n_0^2=\frac{c}{m}$; c , m are the stiffness and mass of the dynamic absorber, respectively [4];

$$d_{2i}=\int_0^l u_i^2\,dx;\ K_1=\theta_1\big(D_0+f(\zeta_{ot})\big);\ K_2=\theta_2\big(D_0+f(\zeta_{ot})\big);$$
$$H_1=\frac{3EI}{\rho Ad_{2i}p_i^2}\sum_{k=1}^{n}C_kq_{ia}^k\frac{h^k}{2^k(k+3)}\int_0^l u_i\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2u_i}{\partial x^2}\left|\frac{\partial^2u_i}{\partial x^2}\right|^k\right)dx;$$

E is modulus of elasticity; $\eta_1,\eta_2=\eta_{22}sign(\omega)$are constant coefficients that depend on the elastic dissipative properties of the beam material and are determined from the hysteresis surface; ω is frequency; $j^2=-1$; $C_0,C_1,\dots,C_n$ are the experimentally determined parameters of the hysteresis node, which depend on the damping properties of the beam material; ρ, A are the density and cross-sectional area of the beam material, respectively; $I=\frac{bh^3}{12}$; $b=const\ va\ h=const$ are the beam width and height; p_i is the natural frequency of the beam; $u_i(x)$ are the natural vibration modes; $u_{i0}(x_0)$ are the values of the natural vibration modes $x_0$at the location of the dynamic absorber; $\theta_1,\theta_2=\theta_{22}sign(\omega)$ are constant coefficients depending on the elastic dissipative properties of the dynamic absorber material, which are determined from the hysteresis surface; $f(\zeta_{ot})$ are the decrements of the vibrations, ζ_{ot} are functions of the absolute value of the relative deformation,

$$f(\zeta_{ot})=D_1\zeta_{ot}+D_2\zeta_{ot}^2\dots+D_n\zeta_{ot}^n,$$

3. Results and Discussion

$D_0,D_1,\dots,D_n$- are experimentally determined parameters of the hysteresis node and depend on the damping properties of the dynamic absorber material [6].

Obtained equations (1) the invariant points amplitude - frequency flatness according to frequency on the axis coordinates determination possible[7].

$$\omega_{a1,2,3,4}=\pm\frac{p_i}{2}[-[(((K_1-1)(A_0+2)\eta_2-$$
$$-(-1+(A_0+2)K_2)\eta_1)C_0-1+(A_0+2)K_2)-(K_1-1)(A_0+1)\times$$
$$\times(2K_2-1)\frac{n_0^2}{p_i^2}+H_1(((K_1-1)\eta_2-K_2\eta_1)(A_0+2)+\eta_1)]\frac{n_0^2}{p_i^2}\pm$$
$$\pm\sqrt{D_a}]^{\frac{1}{2}}\times\left[(1+(M_0-A_0-2)K_2)\frac{n_0^2}{p_i^2}+\eta_2(C_0+H_1)\right]^{-\frac{1}{2}};$$

$$\omega_{b1,2,3,4}=\pm\frac{p_i}{2}[-[(((K_1-1)(A_0+1)\eta_2+ \qquad (2)$$
$$+(1+A_0K_2)\eta_1)C_0-1-A_0K_2)+((K_1-1)(A_0+1)\times$$
$$\times(2K_2+1)\frac{n_0^2}{p_i^2}+H_1(((K_1-1)\eta_2+K_2\eta_1)A_0+2(K_1-1)\eta_2+\eta_1)]\frac{n_0^2}{p_i^2}\pm$$
$$\pm\sqrt{D_b}]^{\frac{1}{2}}\times\left[(1+(M_0-A_0)K_2)\frac{n_0^2}{p_i^2}+\eta_2(C_0+H_1)\right]^{-\frac{1}{2}},$$

where

$$D_a = \{[(((K_1-1)(A_0+2)\eta_2 -$$
$$-(-1+(A_0+2)K_2)\eta_1)C_0 - 1 + (A_0+2)K_2) - (K_1-1)(A_0+1)\times$$
$$\times(2K_2-1)\frac{n_0^2}{p_i^2} + H_1\left(((K_1-1)\eta_2 - K_2\eta_1)(A_0+2)+\eta_1\right)]^2 -$$
$$-4\left[(1+(M_0-A_0-2)K_2)\frac{n_0^2}{p_i^2} + \eta_2(C_0+H_1)\right](A_0+1)\times$$
$$\times[\left(C_0\eta_2((K_1-1)^2+K_2^2+K_2) - (2K_2-1)(C_0\eta_1-1)(K_1-1)\right)+$$
$$+(\eta_2((K_1-1)^2+K_2^2+K_2) - \eta_1(2K_2-1)(K_1-1))H_1]\}\frac{n_0^4}{p_i^4};$$
$$D_b = \{[(((K_1-1)(A_0+1)\eta_2 +$$
$$+(1+A_0K_2)\eta_1)C_0 - 1 - A_0K_2) + ((K_1-1)(A_0+1)\times$$
$$\times(2K_2+1)\frac{n_0^2}{p_i^2} + H_1\left(((K_1-1)\eta_2 + K_2\eta_1)A_0 + 2(K_1-1)\eta_2+\eta_1\right)]^2 -$$
$$-4\left[(1+(M_0-A_0)K_2)\frac{n_0^2}{p_i^2} + \eta_2(C_0+H_1)\right](A_0+1)[(C_0+H_1)\times$$
$$\times((K_1-1)^2 - K_2^2 - K_2)\eta_2 + (C_0\eta_1 - 1 + \eta_1H_1)(K_1-1)]\}\frac{n_0^4}{p_i^4}.$$

Determined (2) based on the frequencies and the absolute value of the transfer function, the following relations can be obtained for the system under consideration[8]:

$$\frac{\left(B_{1a,1}+u_i\omega_{a,1}^2A_{1a,1}\right)^2+\left(B_{2a,1}+u_i\omega_{a,1}^2A_{2a,1}\right)^2}{B_{1a,1}^2+B_{2a,1}^2} -$$
$$-\frac{\left(B_{1a,2}+u_i\omega_{a,2}^2A_{1a,2}\right)^2+\left(B_{2a,2}+u_i\omega_{a,2}^2A_{2a,2}\right)^2}{B_{1a,2}^2+B_{2a,2}^2} = 0;$$
$$\frac{\left(B_{1b,1}+u_i\omega_{b,1}^2A_{1b,1}\right)^2+\left(B_{2b,1}+u_i\omega_{b,1}^2A_{2b,1}\right)^2}{B_{1b,1}^2+B_{2b,1}^2} -$$
$$-\frac{\left(B_{1b,2}+u_i\omega_{b,2}^2A_{1b,2}\right)^2+\left(B_{2b,2}+u_i\omega_{b,2}^2A_{2b,2}\right)^2}{B_{1b,2}^2+B_{2b,2}^2} = 0, \tag{3}$$

where

$$A_{1a,1} = A_1(\omega_{a,1}^2) = -\omega_{a,1}^2 + (1+d_i\mu\mu_0u_{i0})n_0^2(1-\theta_1(D_0+f(\zeta_{ot}));$$
$$A_{1b,1} = A_1(\omega_{b,1}^2) = -\omega_{b,1}^2 + (1+d_i\mu\mu_0u_{i0})n_0^2(1-\theta_1(D_0+f(\zeta_{ot}));$$
$$A_{2a,1} = A_2(\omega_{a,1}^2) = A_2 = (1+d_i\mu\mu_0u_{i0})n_0^2\theta_2(D_0+f(\zeta_{ot}));$$
$$A_{2b,1} = A_2(\omega_{b,1}^2) = A_2 = (1+d_i\mu\mu_0u_{i0})n_0^2\theta_2(D_0+f(\zeta_{ot}));$$
$$A_{1a,2} = A_1(\omega_{a,2}^2) = -\omega_{a,2}^2 + (1+d_i\mu\mu_0u_{i0})n_0^2(1-\theta_1(D_0+f(\zeta_{ot}));$$
$$A_{1b,2} = A_1(\omega_{b,2}^2) = -\omega_{b,2}^2 + (1+d_i\mu\mu_0u_{i0})n_0^2(1-\theta_1(D_0+f(\zeta_{ot}));$$
$$A_{2a,2} = A_2(\omega_{a,2}^2) = A_2 = (1+d_i\mu\mu_0u_{i0})n_0^2\theta_2(D_0+f(\zeta_{ot}));$$
$$A_{2b,2} = A_2(\omega_{b,2}^2) = A_2 = (1+d_i\mu\mu_0u_{i0})n_0^2\theta_2(D_0+f(\zeta_{ot}));$$
$$B_{1a,1} = B_1(\omega_{a,1}^2) = [-\omega_{a,1}^2 + (1-\eta_1C_0)p_i^2 - \eta_1\frac{3EI}{\rho Ad_{2i}}\times$$
$$\times\sum_{k=1}^{n}C_kq_{ia}^k\frac{h^k}{2^k(k+3)}\int_0^l u_i\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2u_i}{\partial x^2}\left|\frac{\partial^2u_i}{\partial x^2}\right|^k\right)dx]\times[-\omega_{a,1}^2+$$
$$+n_0^2\left(1-\theta_1\left(D_0+f(\zeta_{ot})\right)\right)] - \eta_2[C_0p_i^2+\frac{3EI}{\rho Ad_{2i}}\times$$
$$\times\sum_{k=1}^{n}C_kq_{ia}^k\frac{h^k}{2^k(k+3)}\int_0^l u_i\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2u_i}{\partial x^2}\left|\frac{\partial^2u_i}{\partial x^2}\right|^k\right)dx]\times$$
$$\times n_0^2\left(1+\theta_2\left(D_0+f(\zeta_{ot})\right)\right) - \mu\mu_0n_0^2u_{i0}^2\omega_{a,1}^2(1-\theta_1(D_0+f(\zeta_{ot}));$$
$$B_{1b,1} = B_1(\omega_{b,1}^2) = [-\omega_{b,1}^2 + (1-\eta_1C_0)p_i^2 - \eta_1\frac{3EI}{\rho Ad_{2i}}\times$$

$$\times \sum_{k=1}^{n} C_k q_{ia}^k \frac{h^k}{2^k(k+3)} \int_0^l u_i \frac{\partial^2}{\partial x^2} (\frac{\partial^2 u_i}{\partial x^2} \left|\frac{\partial^2 u_i}{\partial x^2}\right|^k) dx] \times [-\omega_{b,1}^2 +$$

$$+n_0^2 \left(1 - \theta_1\left(D_0 + f(\zeta_{ot})\right)\right)] - \eta_2 [C_0 p_i^2 + \frac{3EI}{\rho A d_{2i}} \times$$

$$\times \sum_{k=1}^{n} C_k q_{ia}^k \frac{h^k}{2^k(k+3)} \int_0^l u_i \frac{\partial^2}{\partial x^2} (\frac{\partial^2 u_i}{\partial x^2} \left|\frac{\partial^2 u_i}{\partial x^2}\right|^k) dx] \times$$

$$\times n_0^2 \left(1 + \theta_2\left(D_0 + f(\zeta_{ot})\right)\right) - \mu\mu_0 n_0^2 u_{i0}^2 \omega_{b,1}^2 (1 - \theta_1\left(D_0 + f(\zeta_{ot})\right));$$

$$B_{2a,1} = B_2\left(\omega_{a,1}^2\right) = [-\omega_{a,1}^2 + (1 - \eta_1 C_0) p_i^2 - \eta_1 \frac{3EI}{\rho A d_{2i}} \times$$

$$\times \sum_{k=1}^{n} C_k q_{ia}^k \frac{h^k}{2^k(k+3)} \int_0^l u_i \frac{\partial^2}{\partial x^2} (\frac{\partial^2 u_i}{\partial x^2} \left|\frac{\partial^2 u_i}{\partial x^2}\right|^k) dx] \times$$

$$\times n_0^2 \left(1 + \theta_2\left(D_0 + f(\zeta_{ot})\right)\right) + \eta_2 [C_0 p_i^2 + \frac{3EI}{\rho A d_{2i}} \times$$

$$\times \sum_{k=1}^{n} C_k q_{ia}^k \frac{h^k}{2^k(k+3)} \int_0^l u_i \frac{\partial^2}{\partial x^2} (\frac{\partial^2 u_i}{\partial x^2} \left|\frac{\partial^2 u_i}{\partial x^2}\right|^k) dx] \times$$

$$\times \left[-\omega_{a,1}^2 + n_0^2 \left(1 - \theta_1\left(D_0 + f(\zeta_{ot})\right)\right)\right] - \mu\mu_0 n_0^2 u_{i0}^2 \theta_2\left(D_0 + f(\zeta_{ot})\right)) \omega_{a,1}^2;$$

$$B_{2b,1} = B_2\left(\omega_{b,1}^2\right) = [-\omega_{b,1}^2 + (1 - \eta_1 C_0) p_i^2 - \eta_1 \frac{3EI}{\rho A d_{2i}} \times$$

$$\times \sum_{k=1}^{n} C_k q_{ia}^k \frac{h^k}{2^k(k+3)} \int_0^l u_i \frac{\partial^2}{\partial x^2} (\frac{\partial^2 u_i}{\partial x^2} \left|\frac{\partial^2 u_i}{\partial x^2}\right|^k) dx] \times$$

$$\times n_0^2 \left(1 + \theta_2\left(D_0 + f(\zeta_{ot})\right)\right) + \eta_2 [C_0 p_i^2 + \frac{3EI}{\rho A d_{2i}} \times$$

$$\times \sum_{k=1}^{n} C_k q_{ia}^k \frac{h^k}{2^k(k+3)} \int_0^l u_i \frac{\partial^2}{\partial x^2} (\frac{\partial^2 u_i}{\partial x^2} \left|\frac{\partial^2 u_i}{\partial x^2}\right|^k) dx] \times$$

$$\times \left[-\omega_{b,1}^2 + n_0^2 \left(1 - \theta_1\left(D_0 + f(\zeta_{ot})\right)\right)\right] - \mu\mu_0 n_0^2 u_{i0}^2 \theta_2\left(D_0 + f(\zeta_{ot})\right)) \omega_{b,1}^2;$$

$$B_{1a,2} = B_1\left(\omega_{a,2}^2\right) = [-\omega_{a,2}^2 + (1 - \eta_1 C_0) p_i^2 - \eta_1 \frac{3EI}{\rho A d_{2i}} \times$$

$$\times \sum_{k=1}^{n} C_k q_{ia}^k \frac{h^k}{2^k(k+3)} \int_0^l u_i \frac{\partial^2}{\partial x^2} (\frac{\partial^2 u_i}{\partial x^2} \left|\frac{\partial^2 u_i}{\partial x^2}\right|^k) dx] \times [-\omega_{a,2}^2 +$$

$$+n_0^2 \left(1 - \theta_1\left(D_0 + f(\zeta_{ot})\right)\right)] - \eta_2 [C_0 p_i^2 + \frac{3EI}{\rho A d_{2i}} \times$$

$$\times \sum_{k=1}^{n} C_k q_{ia}^k \frac{h^k}{2^k(k+3)} \int_0^l u_i \frac{\partial^2}{\partial x^2} (\frac{\partial^2 u_i}{\partial x^2} \left|\frac{\partial^2 u_i}{\partial x^2}\right|^k) dx] \times$$

$$\times n_0^2 \left(1 + \theta_2\left(D_0 + f(\zeta_{ot})\right)\right) - \mu\mu_0 n_0^2 u_{i0}^2 \omega_{a,2}^2 (1 - \theta_1\left(D_0 + f(\zeta_{ot})\right));$$

$$B_{1b,2} = B_1\left(\omega_{b,2}^2\right) = [-\omega_{b,2}^2 + (1 - \eta_1 C_0) p_i^2 - \eta_1 \frac{3EI}{\rho A d_{2i}} \times$$

$$\times \sum_{k=1}^{n} C_k q_{ia}^k \frac{h^k}{2^k(k+3)} \int_0^l u_i \frac{\partial^2}{\partial x^2} (\frac{\partial^2 u_i}{\partial x^2} \left|\frac{\partial^2 u_i}{\partial x^2}\right|^k) dx] \times [-\omega_{b,2}^2 +$$

$$+n_0^2 \left(1 - \theta_1\left(D_0 + f(\zeta_{ot})\right)\right)] - \eta_2 [C_0 p_i^2 + \frac{3EI}{\rho A d_{2i}} \times$$

$$\times \sum_{k=1}^{n} C_k q_{ia}^k \frac{h^k}{2^k(k+3)} \int_0^l u_i \frac{\partial^2}{\partial x^2} (\frac{\partial^2 u_i}{\partial x^2} \left|\frac{\partial^2 u_i}{\partial x^2}\right|^k) dx] \times$$

$$\times n_0^2\left(1+\theta_2\left(D_0+f(\zeta_{ot})\right)\right)-\mu\mu_0 n_0^2 u_{i0}^2\omega_{b,2}^2(1-\theta_1\left(D_0+f(\zeta_{ot})\right));$$

$$B_{2a,2}=B_2\left(\omega_{a,2}^2\right)=[-\omega_{a,2}^2+(1-\eta_1 C_0)p_i^2-\eta_1\frac{3EI}{\rho A d_{2i}}\times$$

$$\times\sum_{k=1}^{n}C_k q_{ia}^k\frac{h^k}{2^k(k+3)}\int_0^l u_i\frac{\partial^2}{\partial x^2}(\frac{\partial^2 u_i}{\partial x^2}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k)dx]\times$$

$$\times n_0^2\left(1+\theta_2\left(D_0+f(\zeta_{ot})\right)\right)+\eta_2[C_0 p_i^2+\frac{3EI}{\rho A d_{2i}}\times$$

$$\times\sum_{k=1}^{n}C_k q_{ia}^k\frac{h^k}{2^k(k+3)}\int_0^l u_i\frac{\partial^2}{\partial x^2}(\frac{\partial^2 u_i}{\partial x^2}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k)dx]\times$$

$$\times\left[-\omega_{a,2}^2+n_0^2\left(1-\theta_1\left(D_0+f(\zeta_{ot})\right)\right)\right]-\mu\mu_0 n_0^2 u_{i0}^2\theta_2\left(D_0+f(\zeta_{ot})\right))\omega_{a,2}^2;$$

$$B_{2b,2}=B_2\left(\omega_{b,2}^2\right)=[-\omega_{b,2}^2+(1-\eta_1 C_0)p_i^2-\eta_1\frac{3EI}{\rho A d_{2i}}\times$$

$$\times\sum_{k=1}^{n}C_k q_{ia}^k\frac{h^k}{2^k(k+3)}\int_0^l u_i\frac{\partial^2}{\partial x^2}(\frac{\partial^2 u_i}{\partial x^2}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k)dx]\times$$

$$\times n_0^2\left(1+\theta_2\left(D_0+f(\zeta_{ot})\right)\right)+\eta_2[C_0 p_i^2+\frac{3EI}{\rho A d_{2i}}\times$$

$$\times\sum_{k=1}^{n}C_k q_{ia}^k\frac{h^k}{2^k(k+3)}\int_0^l u_i\frac{\partial^2}{\partial x^2}(\frac{\partial^2 u_i}{\partial x^2}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k)dx]\times$$

$$\times\left[-\omega_{b,2}^2+n_0^2\left(1-\theta_1\left(D_0+f(\zeta_{ot})\right)\right)\right]-\mu\mu_0 n_0^2 u_{i0}^2\theta_2\left(D_0+f(\zeta_{ot})\right))\omega_{b,2}^2.$$

Determined equations (3) are expressions that do not depend on ω the frequency of the system, and are the interdependence equations of the remaining parameters, and the values of the parameters that satisfy these equations are the optimal values for the system under consideration.

We will analyze the obtained results numerically. In this beam with clamped ends for frequency equation and private vibration uniforms we will get [9].

Beam material for 40X steel from the material we use .

$E=2.08\cdot 10^{11}\ {}^{N}/_{m^2}, \rho=7810\ {}^{kg}/_{m^3}$.

It's dimensions as follows we get: $A=a\cdot h=0.02\,m\cdot 0.004\,m=8\cdot 10^{-5}m^2; l=0.5\,m$.

Let's assume that the moving dynamic absorber $v=0.125\ \frac{m}{s}$moving at linear speed Let it be [10]. Then $t=1;2\ s$ we analyze the amplitude-frequency characteristics of the system under consideration.

The experimentally determined parameters of the hysteresis node of the material as follows we get [11].

$$C_1=6.760624;\ C_2=-8278.5937; C_3=5894761;$$

$\eta_1=\frac{3}{4}; \eta_2=\frac{1}{\pi}$.

Moving dynamic absorber element for the following acceptance we do:

$$\theta_1=\theta_2=0, D_0=0, D_1=0, D_2=0.$$

The rest parameters as follows will be:

$$d_1=0.3183098862; d_2=0.25; d_i=1.273239545; \mu=0.1; \mu_0=2.0$$

$$u_{i0}=u_{10}=1; I=1.066666667\cdot 10^{-10}m^4; \varepsilon p_0=0.001\,m.$$

$t=1\,s$ moving when dynamic absorber beam $x_0=0.125\,m$ at the point will be. Above cited of parameters values based on system amplitude-frequency characteristic graph Let's draw [12] (Fig 1).

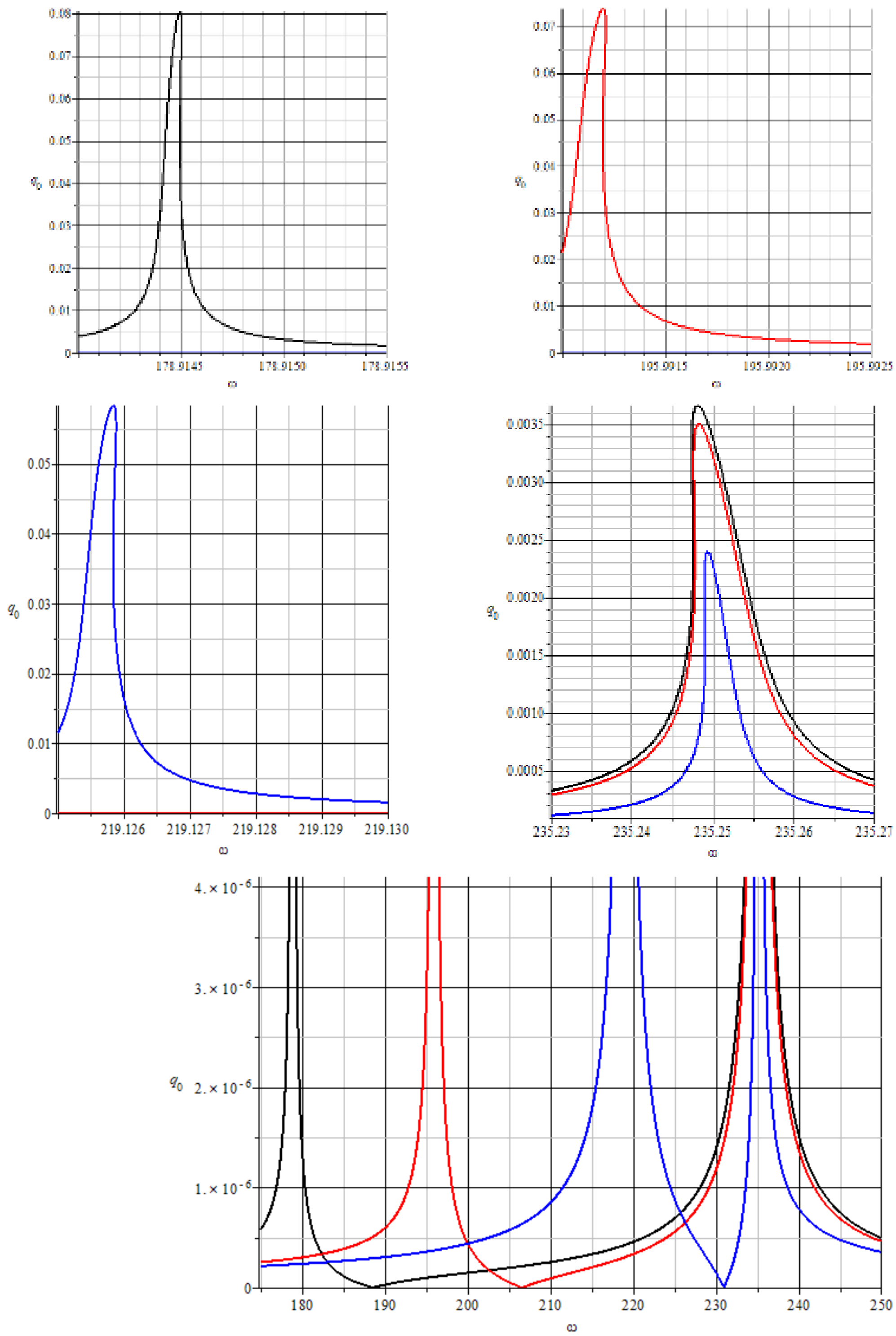
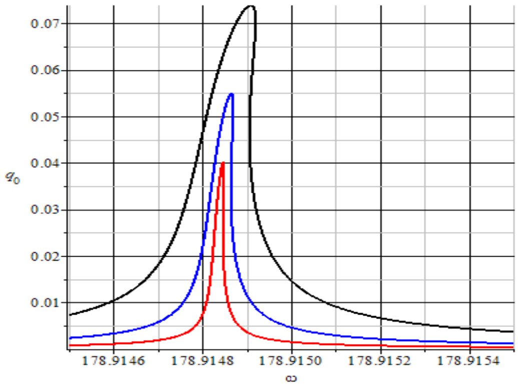


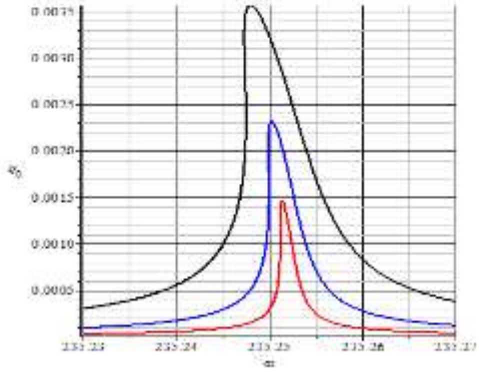
Figure 1. *Amplitude-frequency characteristic ($x_0 = 0.125\, m$).*

In Fig. 1, hysteresis type elastic dissipative characteristic beam at the point $x_0 = 0.125\, m$ was moving dynamic absorber with joint transverse vibrations amplitude-frequency characteristic dynamic damping unit related without change [13]. These graphs are given below. this to say It is possible that the dynamic damping is shooting with his/her frequency beam private frequency approaches and resonance curve line frequency arrow along from the left to the right shifts ($c = 10 \cdot 10^2 \frac{N}{m}$(black), $c = 12 \cdot 10^2 \frac{N}{m}$(red), $c = 15 \cdot 10^2 \frac{N}{m}$(blue)). At these values of the binarity, the largest values of the amplitudes are around the resonant frequency, respectively $q_0 = 0.0036; 0.0035; 0.0024\, m$. From this,

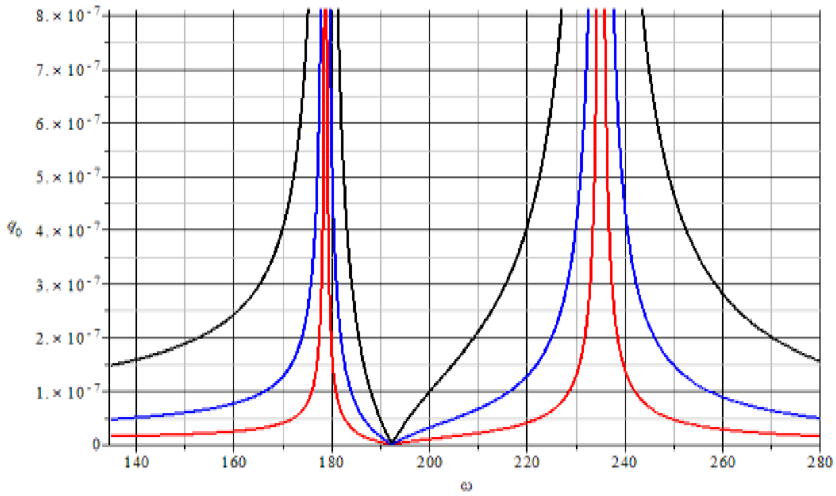
$x_0 = 0.125\,m$for the point of parameters above in values $c = 15 \cdot 10^2 \frac{N}{m}$value religion friend absorber The optimal value of virginity said conclusion to say possible [14] (Fig 2).



a)



b)



c)

Figure 2. *Amplitude-frequency characteristic basis of migration amplitude to the value related without change .*

In Fig. 2, $c = 10 \cdot 10^2 \frac{N}{m}$and at the above values of the parameters beam $x_0 = 0.25\,m$ point amplitude-frequency characteristic basis of migration amplitude to the value related without change depicted ($\varepsilon p_0 = 10^{-2}; 10^{-2.5}; 10^{-3}\,m$ (black; red; blue)) [15]*. From* these graphs this conclusion to do maybe the basis of migration $\varepsilon p_0 = 10^{-2}; 10^{-3}\,m$in values vibration amplitudes the most big values basis of migration $\varepsilon p_0 = 10^{-2.5}\,m$ worth vibration amplitude the most big from the value big will be . Basis of migration amplitude value increase vibration amplitudes to increase take comes , but synchronous in a way to increase take It doesn't come [16]. From these graphs again this to say maybe the basis of migration amplitude value increase unstable amplitudes field also leading to expansion is coming [17].

4. Conclusion

In this study, the optimal parameters of a dynamic absorber designed to minimize the vibrations of a beam with a moving absorber have been analyzed. The investigation demonstrates that the dynamic absorber can significantly reduce the amplitude of vibrations when its parameters are properly optimized. By applying analytical and numerical methods, the relationships between absorber mass ratio, stiffness, and damping coefficients were derived, allowing for precise adjustment of the absorber to achieve the best vibration suppression.

The results confirm that the optimal tuning of the absorber parameters depends not only on the structural characteristics of the beam but also on the dynamic motion of the absorber along the beam's length. The developed mathematical model effectively captures the interaction between the moving absorber and the host structure, which provides a solid foundation for further research and practical applications.

Additionally, the findings suggest that adaptive control of the absorber's parameters could improve vibration suppression in systems subjected to varying external excitations. The proposed optimization approach can be effectively used in engineering designs of bridges, robotic arms, and mechanical structures where vibration reduction is crucial for operational stability and durability.

Future research should focus on the implementation of smart materials and real-time control algorithms to develop self-tuning dynamic absorbers capable of automatically adjusting to changing load conditions. Such advancements could enhance the reliability, efficiency, and lifespan of dynamically loaded beam structures.

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