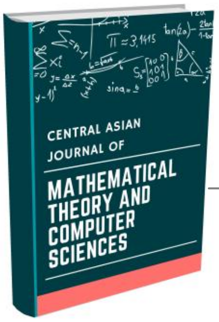

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Mathematical Models the Outflow of a Jet of a Dispersed Mixture into a Flooded Space

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ABSTRACT

The problem of a submerged jet of a mixture of viscous liquids occurs in various jet apparatuses used in agriculture, industrial technology, and fire-fighting installations. The academicians H. A. Rakhmatullin [1] developed the theory of motion of interpenetrating dispersed mixtures of viscous liquids. Mathematical models of a submerged jet a steady axisymmetric, incompressible liquid of constant concentration is considered. For the hydrodynamic model of the liquid mixture, we take the Landau-Rakhmatullin model. The paper develops mathematical models of round jets from a short thin slit in a flooded space.

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Introduction. Let us consider the problem of the outflow of a mixture of viscous fluids in the direction of the *Ox* axis from a sufficiently thin slot located on a vertical plane with a finite amount of motion (momentum).

In various jet devices used in agriculture, industry and fire-fighting installations, the problem of a submerged jet of a mixture of viscous liquids is encountered. The theory of motions of interpenetrating mixtures of viscous fluids was developed by Kh. A. Rakhmatullin [1]. In a number of self-similar jet problems of a single-phase jet of a mixture are considered, which are mainly used for far wakes from a source [2]. In solutions of a number of self-similar problems for a mixture of viscous liquids are given [3]. Below we consider the problem of a submerged jet for a flow region in a close wake, in an axisymmetric formulation. In this problem was solved in a flat setting [4].

The problem is formulated as follows: from an infinitely thin slit located at the point o, the center of the vertical plane perpendicular in the direction of the *Ox* axis, a jet of a mixture of two viscous liquids beats with infinite velocity, but with some finite amount of motion, or momentum I_0. In this case, when the pulse is finite in an infinitely narrow slot, the flow rate through it should be zero (Fig. 1).

Physically, this means that if the gap is very small and the outflow velocity is high, then the pulse can be finite only at very low flow rates.

Main part. Something similar is observed in a variety of inkjet devices. The flow will be symmetric about the Ox axis. From the physical meaning of the problem it follows that, что $\frac{dp}{dx} = 0$.

The equations of motion of the mixture and the equations of continuity take the form [1;3].

$$\left.\begin{aligned}
&u_1\frac{\partial u_1}{\partial x}+v_1\frac{\partial u_1}{\partial y}=\nu_1\frac{1}{y^k}\frac{\partial}{\partial y}\left(y^k\frac{\partial y_1}{\partial y}f_2\right)+\frac{K}{\rho_1}(u_2-u_1)\\
&u_2\frac{\partial u_2}{\partial x}+v_2\frac{\partial u_2}{\partial y}=\nu_1\frac{1}{y^k}\frac{\partial}{\partial y}\left(y^\kappa f_2\frac{\partial y_2}{\partial y}\right)+\frac{K}{\rho_2}(u_1-u_2)\\
&\frac{\partial u_1}{\partial x}+\frac{\partial v_1}{\partial y}=0,\quad \frac{\partial u_2}{\partial x}+\frac{\partial v_2}{\partial y}=0
\end{aligned}\right\}$$

(1)

The phase concentrations are considered constant, where un, vn are the components of the velocity vector; for k = 0, k = 1, we obtain a plane and axisymmetric problem. The boundary conditions on the jet axis due to symmetry will be:

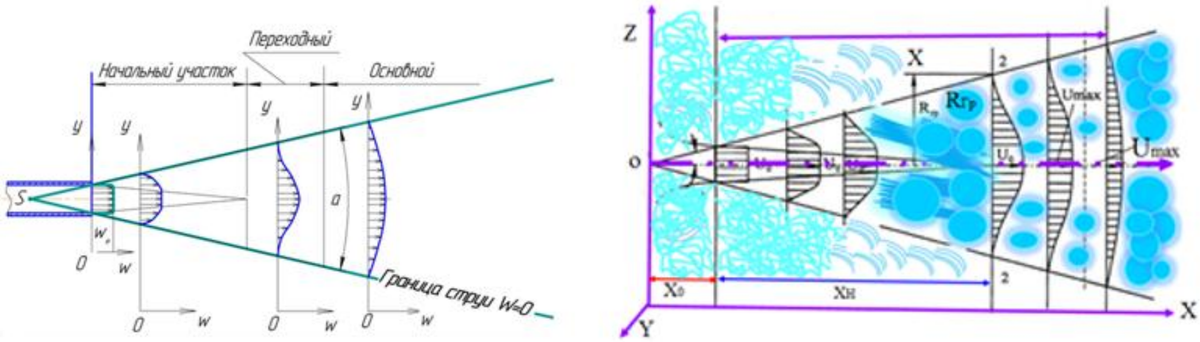
$$v_n=0,\quad \frac{du_i}{\partial y}=0\quad npu\quad y=0,\quad n=1,2,$$

(2)

and on the outer boundary of the jet upon transition to a stationary fluid:

$$u_i=0\quad npu\quad y=\pm\infty.$$

From equations (1), taking into account the boundary conditions, we obtain the integral ratio of the pocket for each phase of the mixture.



Picture.1.Graph and scheme of the flow of the dispersion mixture through the narrow hole of the vertically located vessel into the suppressed space.

The momentum of motion of a mixture of viscous fluids flowing through an arbitrary cross section of the jet will be constant along the axis of symmetry:

$$I=\int_{-\infty}^{\infty}\rho_1u_1^2dy+\int_{-\infty}^{\infty}\rho_2u_2^2dy=f_1\int_{-\infty}^{\infty}\rho_{01}u_1^2dy+f_2\int_{-\infty}^{\infty}\rho_{02}u_2^2dy=I_0=const$$

(3)

If in (1) we introduce the stream function for each phase of the mixture, then the problem is formulated as follows:

$$\left.\begin{aligned}
&\frac{\partial\psi_1}{\partial y}\frac{\partial^2\psi_1}{\partial x\partial y}-\frac{\partial\psi_1}{\partial x}\frac{\partial^2\psi_1}{dy^2}=\nu_1\frac{1}{y^\kappa}\frac{\partial}{\partial y}\left(f_1y^\kappa\frac{\partial^2\psi_1}{\partial y^2}\right)+\frac{K}{\rho_1}\left(\frac{\partial\psi_2}{\partial y}-\frac{\partial\psi_1}{\partial y}\right)\\
&\frac{\partial\psi_2}{\partial y}\frac{\partial^2\psi_2}{\partial x\partial y}-\frac{\partial\psi_2}{\partial x}\frac{\partial^2\psi_2}{\partial y^2}=\nu_2\frac{1}{y^\kappa}\frac{\partial}{\partial y}\left(f_2y^\kappa\frac{\partial^2\psi_2}{\partial y^2}\right)+\frac{K}{\rho_2}\left(\frac{\partial\psi_1}{\partial y}-\frac{\partial\psi_2}{\partial y}\right)
\end{aligned}\right\}$$

(4)

Let us bring equation (1) to the form:

$$\hat{u}_n\frac{\partial\hat{u}_n}{\partial\hat{x}}+\hat{v}_n\frac{\partial\hat{u}_n}{\partial\hat{y}}=(\hat{u}_p-\hat{u}_n)\hat{\kappa}+\frac{\gamma_n}{\gamma_1}\cdot\frac{1}{\hat{y}\kappa}\frac{\partial}{\partial\hat{y}}\left(f_n\hat{y}\kappa\frac{\partial\hat{u}_n}{\partial\hat{y}}\right)\frac{\partial\hat{u}_n\hat{y}^\kappa f_n}{\partial\hat{x}}+\frac{\partial\hat{u}_n\hat{y}^\kappa\hat{v}_n}{\partial\hat{y}}=0$$

(5)

Having chosen dimensionless coordinates,

$\psi_n=\gamma_1\hat{\psi}_n,\ x=(\gamma_1/u_{10})\hat{x},\ y=(\gamma_1/u_0)\hat{y}$

$$\xi = \frac{J_0}{\rho_1 \nu_1^2} x, \quad \eta = \sqrt[3]{\frac{J_0}{\rho_1 \nu_1^2 x^2}} y \qquad (6)$$

Taking a new function and introducing a stream function:

$$\psi_n = x^{\alpha} \varphi_n(\xi, \eta) \qquad (7)$$

$$\frac{8-6k}{3}\eta^2\left(\frac{\partial \psi_n}{\partial \eta}\right)^2 + \xi\eta^2\left[\frac{\partial \varphi_n}{\partial \eta}\frac{\partial^2 \varphi_n}{\partial \eta \partial \xi} - \frac{\partial \varphi_n}{\partial \xi}\frac{\partial^2 \varphi}{\partial \eta^2}\right] - \frac{5-3k}{3}\varphi_n \frac{\partial^2 \varphi_n}{\partial \eta^2} + k\frac{5-3k}{3}\varphi_n \frac{\partial \varphi_n}{\partial \eta}$$

$$-k\xi\eta \frac{\partial \varphi_n}{\partial \xi}\frac{\partial \varphi_n}{\partial \eta} = A_0^{-\frac{k-1}{3}} f_n \frac{\nu_n}{\nu_1}\left[k\eta \frac{\partial \varphi_n}{\partial \eta} + \right.$$

$$\left. + (k+4)\eta^2 \frac{\partial^2 \varphi_n}{\partial \eta^2} + \eta^3 \frac{\partial^3 \varphi_n}{\partial \eta^3}\right] + k^* A_0^{\frac{k+1}{3}} f_n^2 \eta^{k+2}\left(\frac{\partial \varphi_p}{\partial \eta} - \frac{\partial \varphi_n}{\partial \eta}\right) \qquad (8)$$

Where $A_0 = \dfrac{2\pi \hat{I}_0}{1 + \dfrac{\rho_{2i}}{\rho_{1i}}\dfrac{f_2}{f_1}\dfrac{\nu_2^2}{\nu_1^2}}$, , $k_n^* = \dfrac{k_{0n}}{\rho_{ni}} f_n$,

$$\hat{I}_0 = \int_0^{\infty}\left(\hat{u}_1^2 + \frac{\rho_{2i}}{\rho 1i}\frac{f_2}{f_1}\hat{u}_2^2\right)\hat{y}^k d\hat{y},$$

$$J_0 = (2\pi)^k \rho_{1i} f_1 u_{1o}^2 \left(\frac{\nu_1}{\nu_{10}}\right)^{k+1}\hat{I}_0,$$

We seek the solution of system (8) in the form of a series

And taking into account the condition of the self-similar problem, at, we have the following system of equations:

$$\varphi_n(\xi,\eta) = F_{0n}(\eta) + \sum_{m=0}^{\infty} \xi^m F_{mn}(\eta) \qquad (9)$$

Which leads to the following systems of ordinary differential equations for the n-th phase:

$$\frac{8-6k^2}{3}\eta^2 [F'_{0n}]^2 + \frac{5k-3k^2}{3}F_{0n}F'_{0n} - \frac{5-3k}{3}F_{0n}F''_{0n} =$$

$$= A_0^{-\frac{k-1}{3}} f_n \frac{\nu_n}{\nu_1}(\eta F'_{0n} + 4F''_{0n}\eta^{k+2} + \eta^{k+2}F'''_{0n} + k * A_0^{-\frac{k+1}{3}}(F'_{0p}$$

$$2\frac{8-6k^2}{3}\eta^2 \sum_{\ell=0}^{s} F'_{\ell n}F'_{s-\ell,n} + \frac{5k-3k^2}{3}F_{0n}F'_{sn} + \frac{5k-3k^2}{3}F'_{sn} + \frac{5k+3k^2}{3}\sum_{\ell=0}^{\infty} F_{\ell n}F'_{\ell-s,n} =$$

$$= A_0^{-\frac{k-1}{3}} f_n \frac{\nu_n}{\nu_1}\left[\eta^k F'_{sn} + (4+k)\eta^{k+1}F''_{sn} + \eta^{k+2}F'''_{sn}\right] + k * A_0^{-\frac{k+1}{3}} f_n (F'_{sp} - F'_{sn}) \qquad (10)$$

For this system from the boundary conditions (2) we have:

$$\left.\begin{array}{l} F_{nm} = 0, \quad F''_{nm} = 0 \quad npu \quad \eta = 0 \\ F'_{nm} = 0 \quad npu \_\_\_ \eta = \pm\infty \end{array}\right\} \qquad (11)$$

Based on the integral relations, we have

$$\left.\begin{array}{l} 2\int_0^{\infty} f_{01}'^2(\eta)d\eta + 2\dfrac{\rho_{02} f_2}{\rho_{01} f_1}\int_0^{\infty} f_{0,2}'^2(\eta)p\eta = 1 \\ 2\int_0^{\infty} f'_{01}(\eta)f_{11}(\eta)d\eta + 2\dfrac{\rho_{02} f_2}{\rho_{01} f_1}\int_0^{\infty} f'_{02}(\eta)f'_{12}(\eta)d\eta = 0 \\ 2\int_0^{\infty} f'_{01}(\eta)f'_{n1}(\eta)d\eta + 2\dfrac{\rho_{02} f_2}{\rho_{01} f_1}\int_0^{\infty} f'_{02}(\eta)f'_{n2}d\eta = -\bar{\lambda}_{n-1} \end{array}\right\} \qquad (12)$$

Where

$$\bar{\lambda}_{n-1} = \int_0^{\infty}\sum_{k=1}^{n-1} f'_{n-k,1} f'_{k,1} d\eta + \frac{\rho_{02} f_2}{\rho_{01} f_1}\int_0^{\infty}\sum_{k=1}^{n-1} f'_{n-k,2} f'_{k,2} d\eta.$$

Equation Integration $3\frac{\nu_i}{\nu_1} f_{0i}''' + f_{0i} f_{0i}'' + f_{0i}'^2 = 0$

Under boundary conditions

$$f_{0i} = 0, \quad f_{0i}'' = 0 \quad npu \quad \eta = 0;$$
$$f_{0i}' = 0 \quad npu \quad \eta = \infty$$

Gives $f_{0i} = 6ath\frac{\nu_1}{\nu_i} ah$.

The quantity is determined from the condition (3.7.12)

$$a = 0{,}275\left(1 + \frac{s_2\rho_{02}}{s_1\rho_{01}}\right)^{-1/3}.$$

Passing to the old variables, we get

$$\psi_i(x,y) = 6a\sqrt[3]{\frac{J_0\nu_1 x}{\rho_{01}s_1}} th\frac{\nu_1}{\nu_i}\sqrt[3]{\frac{J_0}{\rho_{01}\nu_1^2 s_1}}\frac{ay}{x^{2/3}} \tag{13}$$

The velocity field in the jet is determined by the formulas

$$\left.\begin{aligned} u_1 &= 6a\sqrt[3]{\frac{J_0^2}{\rho_{01}^2 s_1^2 \nu_1 x}}\left(1 - th^2 a\sqrt[3]{\frac{J_0}{\rho_{01}s_1\nu_1^2}}\frac{y}{x^{2/3}}\right) \\ u_2 &= 6a\frac{\nu_1}{\nu_2}\sqrt[3]{\frac{J_0^2}{\rho_{01}^2 s_1^2 \nu_1 x}}\left(1 - th^2 a\frac{\nu_1}{\nu_2}\sqrt[3]{\frac{J_0}{\rho_{01}s_1\nu_1^2}}\frac{y}{x^{2/3}}\right) \end{aligned}\right\} \tag{14}$$

It can be seen from (14) that the main velocity decreases along the Ox axis.

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