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*Article*

# On Fuzzy Filter Topological Group Spaces

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**Abstract:** The work investigates a new idea in fuzzy topological group spaces that combines fuzzy open and closed filter sets: Mixed Fuzzy Filter Topological Group Spaces. There is a research gap in the study of mixed fuzzy topologies, as prior work concentrated on individual fuzzy topologies. This study investigates properties such as fuzzy continuity and open filter sets by defining and analyzing fuzzy filter topological groups using mathematical structures and proofs. The goal is to introduce a bi-fuzzy filter structure in order to generalize fuzzy topological groupings. The findings show that mixed fuzzy filter topological groups can be formed, which has ramifications for further research into applying this concept to rings and other algebraic structures.

**Keywords:** fuzzy filter, fuzzy topological group, fuzzy nbhd filter, fuzzy open filter set, fuzzy closed filter set, mixed fuzzy filter fuzzy topological group

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## 1. Introduction and Preliminaries

The fuzzy (fy) set is a function $FL: Q \to \{q: 0 \le q \le 1\}$ (Zadeh, 1965). the fuzzy Topological (fy Topolo.) space studied by (Chang, 1968 and Lowen 1976). Fy topol. gp showed by (Foster1980). Several types of fuzzy filters are introduced by many authors [1,2, 3-9, 13, 17-21]. Our gool is to given via fuzzy group (fy gp) a new form of fy filter, we have devoted a family of fy filter sets in a fy gp to be a family of fy open filter of fy identity element $0$ of the fuzzy filter Topological group (fy filter Topol. gp). As an application, a mixed fuzzy filter topological group (Mix. fy filter topol. gp) is formed from two given fy filter topol. gps with the help of fy open filter $\varepsilon_{(FQ)_1}(\varepsilon_{(FQ)_2})$ and the presented the terms $FK_1(FK_2)) -$ interior filter and $FK_1(FK_2)) -$ closure filter.

### Defin. 1.1.1 [10]

Let $Q$ be a fy gp in $G$ endowed with the fy topol. $\varepsilon_{FQ}$. Then $(Q, \varepsilon_{FQ})$ is fy topol. gp iff justified the map

$L : (Q, \varepsilon_{F_Q}) \times (Q, \varepsilon_{F_Q}) \to (Q, \varepsilon_{F_Q})$ , $L(q, p) \to q.p^{-1}$ is fy cont.

### Thm. 1.1.2 [10]

Let $(Q, \varepsilon_{FQ})$ be a fy topol. gp and $q \in Q$. Then the maps

$L_p: (Q, \varepsilon_{FQ}) \to (Q, \varepsilon_{FQ})$, $L(q) = q.p^{-1}$ and $L: (Q, \varepsilon_{F_Q}) \to (Q, \varepsilon_{F_Q})$, $L(q) = p.q.p^{-1}$ are fy homeoms.

**Defin. 1.1.3 [11]**

If $Q$ be a gp with two fy topol. gp $\varepsilon_{(FQ)_1}$and $\varepsilon_{(FQ)_2}$. Then $(Q, \varepsilon_{(FQ)_1}, \varepsilon_{(FQ)_2})$ is known a bi- fy topol. gp space.

**Defin. 1.1.4 [11]**

Let$(Q, \varepsilon_{(FQ)_1}, \varepsilon_{(FQ)_2})$ be a bi- fy topol. gp. The fy Topol. $\varepsilon_{(FQ)_1}(\varepsilon_{(FQ)_2})$ on $Q$ known by the collection $\{K \in I^Q : \exists H \in \varepsilon_{(FQ)_2}\ s.t\ cl_{\varepsilon_{(FQ)_1}}(H) \leq K\}$ of all fy open nbhds of 0 s.t $(Q, \varepsilon_{(FQ)_1}(\varepsilon_{(FQ)_2}))$ be a fy topol. gp is known as a mix. fy topol. gp.

**Defin.1.1.5 [12]**

A fy filter $F$ on $Q \neq 1_0$ is subsets class of $I^Q$ with:

(i) If $F_1, F_2 \in F\ then\ F_1 \wedge F_2 \in F$

(ii) If $F_1 \in F\ \ and\ F_1 \leq\ F_2 then\ F_2 \in F$

(iii) $1_0 \notin F$.

**Ex. 1.1.6 [12]**

A classic example of a fy filter is the set $\{K_q\}$ of all fy nhoods of a fy pt. $q$ in a fy topol. space $Q$ called the fy nhoods filter of $q$.

**Defin. 1.1.7 [12]**

A collection $\{W\} \neq 1_0$ of fy subsets of $I^Q$ is a fy base for some fy filter with

(i) if $W_1, W_2 \in W$ then $W_3 \leq W_1 \wedge W_2$for some $W_3 \in W$

(ii) $\emptyset \notin \{W\}$

$the\ collection\ F = \{F \in I^Q :\ \exists B \in \{W\}\ s.t\ B \leq F\}$ is a fy filter.

**Defin. 1.1.8 [10]**

A fy set $E$ of $Q$ is said to be a fy subgp of $Q$ if for all $q, p \in Q$

$$1)\ E(q.p) \geq\ min\ \{E(q), E(p)\}$$

$$2)\ E(q^{-1}) \geq E(q).$$

**Thm. 1.1.9 [11]**

Let $(Q, \varepsilon_{(FQ)_1}, \varepsilon_{(FQ)_2})$ be any bi-fy topol. gp. If $\varepsilon_{(FQ)_2} < \varepsilon_{(FQ)_1}$, then $\varepsilon_{(FQ)_2} < \varepsilon_{(FQ)_1}(\varepsilon_{(FQ)_2}) < \varepsilon_{(FQ)_1}$

**Thm. 1.1.10 [11]**

If $(Q, \varepsilon_{(FQ)_1})$and $(Q, \varepsilon_{(FQ)_2})$ are fy topols. gp s.t $\varepsilon_{(FQ)_2} \leq \varepsilon_{(FQ)_1}$. Let $F\omega_1, F\omega_2$ be a fy fund. system of fy nbhds of 0. Then $F\omega_1(F\omega_2) = \{K \in I^Q : \exists H \in \varepsilon_{(FQ)_2}\ s.t\ cl_{\varepsilon_{(FQ)_1}}(H) \leq K\}$ is a fy fund. system of fy nbhds of 0.

## 2. Materials and Methods

The current work will be done by utilized to constructive manner, depending on mathematical definitions, constructs and proofs to derive theoretical results.

Fy filter topol. and fy gp will used to constructs and definition of fy filter topol. gp and mix. fy topol. gp. The properties them will be studied, including the existence of fund. Systems of fy open filter and fy $T_{F_0}$ filter topol. space.

The concept of bi-fy topol. gp will be gave, deals with the characteristic of fy filter topol.and fuzzy gp. The mix. of fuzziness of filter topol. gp will be searched, including the properties of fuzzy topology.

## 3. Results and Discussion

### 3.3. Fuzzy Filter topological group

We study fy filter topol. gp and fy open filter of fy filter topol. Gp

**Defin. 3.3.1**

A fy filter $F$ on fy gp $Q$ is a subsets class of $I^Q$ with:

(i) If $F\,K_1, F\,K_2 \in F\ then\ F\,K_1 \wedge F\,K_2 \in F$

(ii) If $F\,K_1 \in F\ \ and\ F\,K_1 \leq\ F\,K_2 then\ F\,K_2 \in F$

(iii) $1_0 \notin F$ .

(iv) $FL: (Q,F) \times (Q,F) \to (Q,F)\ , FL(q) = q.p^{-1}$ is fy filter cont.

**Defin. 3.3.2**

Let $(Q,F\ )$ be a fy filter gp space. A subset $F\,K \in I^Q$ is fy open filter gp (with respect to $F$) iff $\forall q \in F\,K$ and $FH \in F(q)$ then $FH \leq FK$ and $Q - FK$ is said $fy$ closed filter gp set.

**Defin. 3.3.3**

A mapping $FL$ from a fy filter gp space$(Q, F_1\ )$ into fy filter gp space $(Q, F_2\ )$ is fy filter cont. iff $FK \in F_2$ implies $FL^{-1}(FK) \in F_1$.

**Thm. 3.3.4**

Let $(Q, \varepsilon_{FQ})$ be a fy filter gp and $q \in Q$. Then $FL: (Q,F\ ) \to (Q,F\ )\ ,\ \ FL(q) = q^{-1}$ and $FL: (Q,F\ ) \to (Q,F\ )\ ,\ \ FL(q) = p.q.p^{-1}$ are fy homeoms.

**Proof**

(1) Clearly $FL$ is $1-1$ . Since $FL(q) = Q(h)\ = Q(q^{-1}) = Q(q)$ for all $q \in Q$ , $FL(Q)\ =\ Q$. Since $FL^{-1}(q) = q^{-1}$ is fy filter cont., $FL$ is fy open filter. Thus, $FL$ is a fy filter homeom.

(2) Let $Fl_p(q): (Q, \varepsilon_{FQ}) \to (Q, \varepsilon_{FQ}),\ \ FL_p(q) =\ q.p$ be a right translation

and $Fg_p(q): (Q, \varepsilon_{FQ}) \to (Q, \varepsilon_{FQ}),\ Fg_p(q) =\ p.q$ be a left translation. Then

$(Fl_p(FE))(q)\ \geq FE(q.p) = \{FE(q), FE(0)\}\ = FE(q) = FE(q.p^{-1}.p) \geq$ $\{FE(q.p^{-1}), FE(p)\}\ = FE(q.p^{-1}) = FE(q).p\ = FL_p(FE)(q)$. Thus $FL_p(q) = Q$ is onto.

Let $Fl: (Q, \varepsilon_{FQ}) \to (Q, \varepsilon_{FQ}) \times (Q, \varepsilon_{FQ}), Fl(q) = (q,p)$ and $Fg: (Q, \varepsilon_{FQ}) \times (Q, \varepsilon_{FQ}) \to (Q, \varepsilon_{FQ}),\ Fg(q,p) =\ q.p$.Then $(Fg \circ Fl)(q) = Fg\big(Fl(q)\big)\ \ = Fg(rq, p^{-1}) =\ q.p^{-1}\ \ = FL_p(q)$

Since $Fg$ and $Fl$ are fy filter cont. implies $FL_p(q)$is fy filter cont. mapping. Since $FL(q) = q^{-1}$ is a fy filter homeom. Similarly, $FL_p(q)$ is a fy filter homeom. and $FL$ is a composition of $FL_p(q)$and $FL(q) = q^{-1}$. Thus $FL(q)$is a fy filter homeom.

**Thm. 3.3.5**

If $(Q, F)$ be a fy filter gp space and let $\{FK_0\} \leq I^Q$ be a fy open filter (with respect to $F$) then $(Q, \varepsilon_{FQ})$ $is$ a fy topol. gp on Q induced by $F$,where $\varepsilon_{FQ} = 1_0 \vee \{FK_0\}$.

**Proof**

Clearly

**Lemma 3.3.6**

Let $Q$ be a fy topol. gp and let $FK$ be the fy open filter of 0. Then $\forall q \in Q,\ q.FK = FK.q$ is the fy open filter of $q$. Here,

$$q.FK = \{q.B\ , B \in FK\}\ and\ FK.q = \{B.q\ , B \in FK\}$$

**Proof**

Since both $FL(q) = q.p$ and $Fg(q) = p.q$ are fy homeoms. and $FL(0) = Fg(0) = 1$ $q.FK = FL(0) = Fg\ (0) = FK.q$ is the fy open filter of q.

**Thm 3.3.7**

Let $(Q, \varepsilon_{F_Q})$ be a fy topol. gp and $\{FK_0\}$ be an fy open filter of $0, FK(0) > 0$ , then $FK.FK \in \{FK_0\}$.

**Proof**

Let $FK$ be an open fy open of $0, FK(0) > 0$ . by definition of$(Q, \varepsilon_{F_Q})$, then the map $FL : (Q, \varepsilon_{F_Q}) \times (Q, \varepsilon_{F_Q}) \rightarrow (Q, \varepsilon_{F_Q})\ ; FL(q, p) = q.p$ is fy homeom. So $FL^{-1}(FK)$ is fy open filter in $(Q \times Q, \varepsilon_{F_Q} \times \varepsilon_{F_Q})$ . But $FL^{-1}(FK)(0,0) = FK(FL(0,0)) = FK(0.0) = FK(0) > 0$ . Hence, there exists fy open filter set $FK$ with $FK \times FK \subset FL^{-1}(FK)$ s.t $(FK \times FK)(0,0) > 0$, i.e

$\{FK(0), FK(0)\} = FK(0) > 0$ . Thus $FL(\ FK \times FK) = FK.FK$ , implies $FK.FK \in \{FK_0\}$

**Thm 3.3.8**

Let $(Q, \varepsilon_{F_Q})$ be a fy topol. gp. If $q \in \{p : Q(p) = max\ \{Q(h)\},\ \forall h \in Q\}$ and $FK$ is a fy open filter of $0$, then $q.FU$ is a fy open filter of $q$ such that $(q.FK)(p) = max\ \{Q(h)\},\ \forall h \in Q\}$.

**Proof**

Since $FK$ is a fy open filter of 0 and $FK(0) = max\ \{Q(h)\},\ \forall h \in Q\}$ . Let $FL(q): (Q, \varepsilon_{F_Q}) \rightarrow (Q, \varepsilon_{F_Q})$, $FL(q) = q.p$. By Thm 2.2.4, $FL$ is a fy homeom. Thus $q.FK$ is a fy open filter set with $q.FK(q) = FK(q.q^{-1}) = FK(0) = \{Q(q)\}$ , $\forall p \in Q$ .

Thus, $q.FU$ is a fy open filter of $q$

**Thm 3.3.9**

Let $(Q, \varepsilon_{F_Q})$ be a fy topol. gp and let $\{FK_0\}$ be the fy open filter of 0. Then

(1) $\forall FK \in \{FK_0\},\ \exists FH \in FK \Rightarrow FK \wedge FH \in \{FK_0\}$.

(2) If $FK \in \{FK_0\}$ then $FK^{-1} \in \{FK_0\}$

(3) $\forall FK \in \{FK_0\}$ is semmtric

(4) $\forall FK \in \{FK_0\}$ and $q \in Q,\ q^{-1}.FK \in \{FK_0\}$, .

**Proof**

(1) Let $FK, FH \in \{ FK_0\}$. We have $FK \wedge FH \in \varepsilon_{F_Q}$ . Also

$$(FK \wedge FH)(0) = \{FK(0), FH(0)\} > 0$$

Thus $FK \wedge FH \in \{FK_0\}$

(2) Let $FK \in \{FK_0\}$, then there exists $FH \in \{FK_0\}$ s.t $FK(0) > 0$ . Let $FL: Q \to Q, FL(q) = q^{-1}$ , then $FL$ is a fy homeom. implies $FL^{-1}(FH) \in \{FK_0\}$

Also

$FK(q) = FL^{-1}(FH)(q) = FH\big(FL(q)\big) \le FK\big(FL(q)\big) = FK(q^{-1}) = FK^{-1}(q)$ and $FK^{-1} = FK(0^{-1}) = FK(0) > 0$

Thus $FK^{-1} \in \{FK_0\}$

(3) Let $FK \in \{FK_0\}$, then by condition (2) we have $FK^{-1} \in \{FK_0\}$, then there. let $FH = FK \wedge FK^{-1}$ , then$FH = FH^{-1}$ is symmetric.

Also

$FH^{-1}(q) = min\,\{FK(q), FK^{-1}(q)\} \le FK(q)$ and

$$FH^{-1}(0) = min\,\{FK(0), FK^{-1}(0)\} = \{FK(0), FK(0^{-1})\} = \{FK(0), FK(0)\} = FK(0)$$

Thus $FK(0)$ is symmetric

(4) Let $FK(0) \in \{FK(0)_0\}$, then $FK(0) > 0$ . Since $FK$ is a fy open filter of 0. Implies $q^{-1}.FK$is fy open filter of 0

Also

$$q^{-1}.FK(0) = FK(q.0) = FK(0) > 0$$

**Corollary 3.3.10**

Let $\{FW_0\}$ be a fy filter base on $Q$ then,

(1) $\forall FW \in \{FW_0\}$, $\exists FH \in \{FW_0\} \Rightarrow FW \wedge FH \in \{FW_0\}$.

(2) If $FW \in \{FW_0\}$ then $FW^{-1} \in \{FW_0\}$

(3) $\forall FW \in \{FW_0\}$ is semmtric

(4) $\forall FW \in \{FW_0\}$ and $q \in Q$, $q^{-1}.FW \in \{FW_0\}$.

Conversely, there is only fy topol. gp on $Q$ for $FW$ is a fy open filter base at 0.

**Thm 3.3.11**

Let $Q$ be a fy gp and $q \in Q$ be an invertible fy pt s.t $q \in \{p : FK(p) = max\,\{FK(h)\}, \forall h \in Q\}$. Let $\{FK_0\}$ be a fundamental fy open filter of 0 justifying (1) – (4) of Thm (2.2.9), there is only fy topol. gp $\varepsilon_{F_Q}$ on $Q$ where $\varepsilon_{FQ} = 1_0 \vee \{FK_0\}$.

**Proof**

Clearly that $(Q, \varepsilon_{F_Q})$ is a unique fuzzy topol. gp. We claim the following map $FL: (Q, \varepsilon_{F_Q}) \times (Q, \varepsilon_{F_Q}) \longrightarrow (Q, \varepsilon_{F_Q})$ defined by $FL(q, p) = q.p^{-1}$ is a fuzzy Contin.

Now let $FK$ be a fy open filter of $q.p^{-1}$ , then $FL^{-1}(FK)(q, p^{-1}) = FK\big(FL(q.p)\big) = FK(q.p) > 0$ . by Theorem 2.2.7 there exists a fy open filter $FH$ of $p$ s.t $FK.\, FH^{-1} \le FK$

Thus $FL$ is fy Contin.

**Thm 3.3.12**

Let fy open filter $\{FK_0\}$ of 0, A fy topol. gp $(Q, \varepsilon_{F_Q})$ is fy $T_{F_0}$ −topol. gp. space iff $\{0\}$ is fy closed filter set.

**Proof**

Let $(Q, \varepsilon_{F_Q})$ be a $T_{F_0}$ and $\{FK_0\}$ be a fy open filter of 0 For any $q \neq 0$, there is $FH$ , $FH(q) > 0$ s.t $FH(0) = 0$, implies $q \notin \underline{F\{0\}}$. Since $q$ is arbitrary. Thus $\underline{F\{0\}} = \{0_\alpha\}$ and $\{0_\alpha\}$ is fy closed filter set.

Conversely, let $\{FK_0\}$ be a fy open filter of 0 and $\wedge_{FK \in \{FK_0\}} FK = \{0\}$ . Let $q$ and $p$ are fuzzy points with different support. Therefor $q.p^{-1} \neq 0$, so there is $FH \in \{FK_0\}$ , s.t $FH(q.p^{-1}) = 0$. Now by theorem 2.2.4, $p.FK(q)$ is a fy open filter of $p$ and $(p.FK)(q) = FK(q.p^{-1}) = 0$. Thus $q \notin p.FK$ and $(Q, \varepsilon_{F_Q})$ is fy $T_{F_0}$-topol. gp space.

**Corollary 3.3.13**

Let $[FW\}$ be a fy filter base of 0 then $(Q, \varepsilon_{F_Q})$ is fy $T_{F_0}$ −topol. gp. space iff $\{0\}$ is fy closed filter set:

**Theorem 3.3.14**

Let $(Q, \varepsilon_{F_Q})$ be a fy topol. gp. If $q \in \{p : Q(p) = max\,\{Q(h)\}, \forall h \in Q\}$ and $FK$ is a fy open filter of $q$ s.t $FK(q) = max\,\{Q(p)\}, \forall p \in Q\}$, then $FK.q$ is a fy open filter of 0 such that $K(0) = max\,\{Q(p), \forall h \in Q\}$

**Proof**

Since $FK$ is a fy open filter of $q$ and $FK(q) = max\,\{Q(p)\}, \forall p \in Q\}$ . $FL(q): (Q, \varepsilon_{F_Q}) \to (Q, \varepsilon_{F_Q})$, $FL(q) = q.p$. by Thm 1.1.2, $FL$ is a fy homeom. Thus $q^{-1}.FH$ is a fy open filter set.

$$FH.q^{-1}(0) = FH(0.q^{-1}) = FH(q^{-1}) = \{Q(p), \forall p \in Q\} = FH(0)\,.$$

$FH.p^{-1}(q) = FH(q.p) \geq FH(q.p) = FH.p^{-1}(q)$ for all $q \in Q$ .Thus there exists $FH.p^{-1}$ fy open filter set such that $FH.p^{-1} \leq FK.p^{-1}(q)$ and $FH.p^{-1}(0) = FK.p^{-1}(0) = Q$.

## 4.4 Mixed fuzzy Filter topological group

**Defin. 4.4.1**

Let $F_1, F_2$ on a fy gp $Q$. Then the triple $(Q, F_1, F_2)$ is known a bi- fy filter Group space.

**Defin. 4.4.2**

Let$(Q, F_1, F_2)$ be a bi- fy filter gp. The fy filter gp $F_1(F_2)$ on $Q$ by the collection $\{FK \in I^Q: \exists FH \in F_2\; s.t\; cl_{F_1}(FH) \leq FK\}$ of all fy open filter of 0 s.t $(Q, F_1(F_2))$ be a fy filter. gp, is known as a mixed fy filter gp.

**Thm 4.4.3**

Let $(Q, F_1$and$(Q, F_2)$ be two fy filter gp such that $F_1, F_2$. Let $\{FK_1\}, \{FK_2\}$ is a funda. system of fy open filter of 0 in the $F_1, F_2$ respectively. Then $\{FK_1(FK_2)\} = \{FK \in I^Q: \exists FH \in F_2\; s.t\; cl_{F_1}(FH) \leq FK\}$ is a funda. system of fuzzy open filter of 0

**Proof**

We claim that the conditions 1,2,3 and 4 of Theorem 2.2.9 are satisfied by the collection $FK_1(FK_2)$ .

$(i)$ Let $FK_1, FK_2 \in \{FK_2\}$ then there is $FH_1, FH_2 \in \{FK_2\}$ s.t $cl_{F_1}(FH_1) \leq FK_1$ and $cl_{F_1}(FH_2) \leq Fk_2$, we have $FH_1 \wedge FH_2 \in \{FK_2\}$.[Thm2.2.9] and

$cl_{F_1}(FH_1 \wedge FH_2) \leq cl_{F_1}(FH_1) \wedge cl_{F_1}(FH_2) \leq FK_1 \wedge FK_2$ also $(FK_1 \wedge FK_2)(0) = \{FK_1(0), FK_2(0)\} > 0$. Thus, $FK_1 \wedge FK_2 \in \{FK_1(FK_2)\}$,

$(ii)$ Let $FK \in \{FK_1(FK_2)\}$ , then there is $FH \in \{FK_2\}$ s.t $cl_{F_1}(FH) \leq FK$, by Thm 2.2.9

$FH^{-1} \in \{FK_2\}$ . Now $cl_{F_1}(FH^{-1}) = (cl_{F_1}(FH))^{-1} \leq FK^{-1}$ , $FK^{-1}(0) = FK(0^{-1}) = FK(0) > 0$

Thus $FK^{-1} \in \{FK_1(FK_2)\}$,

$(iii)$ Let $FK \in \{FK_1(FK_2)\}$, by above condition $FK^{-1} \in \{FK_1(FK_2)\}$ $-U \in V_\rho$. Let $FK_1 = FK \wedge FK^{-1}$, Now

$FK_1(0) = [FK(0), FK^{-1}(0)\} = [FK(0), FK(0^{-1})\} = [FK(0), FK(0)\} = FK(0) > 0$
$FK(0)_1(q) = [FK(q), FK^{-1}(q)\} = [FK(q), FK(q^{-1})\} = [FK(0), FK(0)\} = FK(0) > 0$. Thus $FK$is symmetric

$(iv)$ Let $FK \in \{FK_1(FK_2)\}$ , then there is $FH \in \{FK_2\}$ s.t $cl_{F_1}(FH) \leq FK$ , and $FH.q \in \{FK_2\}$ . Now, $cl_{F_1}(FH.q) = cl_{F_1}(FH).q \leq FK.q$ , and $(FK.q)(0) = FK(0) > 0$ . Thus, $FK.q \in \{FK_1(FK_2)\}$

**Coro. 4.4.4**

Let $(Q, F_1)$ and$(Q, F_2)$ be two fy filter gp such that $F_1, F_2$. Let $\{FW_1\}, \{FW_2\}$ is fy open filter base of 0 in the $F_1, F_2$ respectively. Then $\{FW_1(FW_2)\} = \{FK \in I^Q : \exists FH \in F_2\ s.t\ cl_{F_1}(FH) \leq FK\}$ is a fy open filter basis of 0. Conversely, there is only fy topol. gp on $Q$ for $\{FW_1(FW_2)\}$.

**Thm 4.4.5**

Let $\{FK_1(FK_2)\} = \{FK \in I^Q : \exists FH \in F_2\ s.t\ cl_{F_1}(FH) \leq FK\}$ be a fundamental system of fuzzy open filter of 0 satisfied the conditions of Thm 3.3.3. then $\exists!$ fy topolo. gp $\varepsilon_{(FQ)_1}(\varepsilon_{(FQ)_2})$ s.t $(Q, \varepsilon_{(FQ)_1}(\varepsilon_{(FQ)_2}))$ is mix. fy topolo. gp, where $\varepsilon_{(FQ)_1}(\varepsilon_{(FQ)_2}) = 1_0 \vee \{FK_1(FK_2)\}$

**Proof**: By the same way of Thm 2.2.10

## 4. Conclusion

This paper first introduces the concept "fy filter topol. gp", These studies give us a link between fy filter topol. and fy gp theory which is the combined form of fy filter topol. and fy gp. This paper reports on the more generalized form of fuzzy topol. gp definition that includes bi fy filter topol. spaces as a special case. The properties of fy filter topol. gp, fuzzy open filter, fuzzy closed filter, fy filter base and mix. fy filter topol.gp are analyzing and consistent with the earlier findings on fy filter topol. spaces and fy algebraic structures too. The work constructs mix. Fy filter topol. gp by using fy open filter to bi-fy filter topol. gp. This work is similar to the mix. fy topol. gp spaces gave by Das [11]. In Das's construction, two fy gp with distinct fy topols. are put together to create a new mix. fy topol. gp space.

If $Q$ be a set and $(Q, F_1, F_2)$ be a bi- fy filter gp Then fy filter topol. gp $\varepsilon_{(FQ)_1}(\varepsilon_{(FQ)_2})$ on $Q$ indicated in this article finding $\varepsilon_{(FQ)_1}(\varepsilon_{(FQ)_2})$ is a mix. fy filter topol. gp. structure also we obtain that for any mix. fy filter on fy gp is a fy filter topol. gp. In future this work can also be extended further on rings and also in other areas.

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