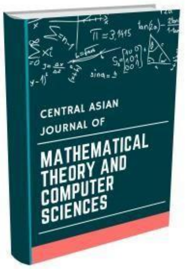

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Article

Middle Arithmetic Mean ($\underline{x}_0$)

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Abstract: In this study, we define the concepts of an arithmetic mean or ordinary arithmetic mean $(\underline{x})$, maximum arithmetic mean $(\underline{x}_{+1})$, minimum arithmetic mean $(\underline{x}_{-1})$, middle arithmetic mean $(\underline{x}_0)$, the numerical difference between two numbers ND , the relative difference between two numbers RD , we compare between the ordinary arithmetic mean $(\underline{x})$ and the middle arithmetic mean $(\underline{x}_0)$ by the value of the relative difference between them and we show that if $x = 1, 2, 3, \dots, n$, then $\underline{x} = \underline{x}_0$.

Keywords: Ordinary Arithmetic Mean, Maximum Arithmetic Mean, Minimum Arithmetic Mean, Middle Arithmetic Mean, The Numerical Difference Between Two Numbers and The Relative Difference Between Two Numbers .

1. Introduction

Arithmetic mean : This type of mean is a simple type of mean and we calculate it by summing up the entries in a set of data divided by the number of entries . Arithmetic mean is shown by the symbol $\underline{x}$. If we have $x_1, x_2, x_3, \dots, x_n$, we calculate the mean of this data by the following formula [1].

$$\underline{x} = \frac{x_1 + x_2 + x_3 \dots + x_n}{n}$$

indicates the number of data. n

Example 1-1 :

Let $x = 1, 2, 3, 4, 5$, then Arithmetic mean (or Ordinary Arithmetic Mean)

$$\underline{x} = \frac{1+2+3+4+5}{5} = \frac{15}{5} = 3$$

1-2 Maximum Arithmetic Mean ($\underline{x}_{+1}$) :

The maximum arithmetic mean is represented by the symbol $\underline{x}_{+1}$, and it can be calculated in the following way :

If $x = x_1, x_2, x_3, \dots, x_n$

Let $\{y_1, y_2, y_3, \dots, y_n\} = \{x_1, x_2, x_3, \dots, x_n\}$

Such that $y_1 < y_2 < y_3 < \cdots < y_n$

Let $a_0 = y_1, a_1 = \frac{y_1+y_2}{2}, a_2 = \frac{a_1+y_3}{2}, a_3 = \frac{a_2+y_4}{2}, \dots, a_{n-1} = \frac{a_{n-2}+y_n}{2}$,

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then

$$\underline{x}_{+1} = a_{n-1} = \frac{a_{n-2} + y_n}{2} \quad , n \in N \ , n \geq 2$$
$$= \frac{y_1}{2^{n-1}} + \frac{y_2}{2^{n-1}} + \frac{y_3}{2^{n-2}} + \cdots + \frac{y_n}{2^{n-(n-1)=1}}$$
$$= \frac{y_1 + y_2 + 2y_3 + 2^2 y_4 + \cdots + 2^{n-2} y_n}{2^{n-1}}$$

1-3 Minimum Arithmetic Mean ($\underline{x}_{-1}$) :

The minimum arithmetic mean is represented by the symbol $\underline{x}_{-1}$, and it can be calculated in the following way :

If $x = x_1 , x_2 , x_3 , \ldots , x_n$

Let $\{z_1 , z_2 , z_3 , \ldots , z_n\} = \{x_1 , x_2 , x_3 , \ldots , x_n\}$

Such that $z_1 > z_2 > z_3 > \cdots > z_n$

Let $b_0 = z_1$

$$, b_1 = \frac{z_1 + z_2}{2} \ , b_2 = \frac{b_1 + z_3}{2} \ , b_3 = \frac{b_2 + z_4}{2} \ , \ldots , b_{n-1} = \frac{b_{n-2} + z_n}{2}$$

then

$$\underline{x}_{-1} = b_{n-1} = \frac{b_{n-2} + z_n}{2} \quad , n \in N , n \geq 2$$
$$= \frac{z_1}{2^{n-1}} + \frac{z_2}{2^{n-1}} + \frac{z_3}{2^{n-2}} + \cdots + \frac{z_n}{2^{n-(n-1)=1}}$$
$$= \frac{z_1 + z_2 + 2z_3 + 2^2 z_4 + \cdots + 2^{n-2} z_n}{2^{n-1}}$$

1-4 Middle Arithmetic Mean ($\underline{x}_0$) :

The middle arithmetic mean is represented by the symbol $\underline{x}_0$, and it can be calculated as follows :

$$\underline{x}_0 = \frac{\underline{x}_{+1} + \underline{x}_{-1}}{2}$$

Example 1-5 :

Let $x = 1 , 0 , 3 , 5 , 2 , 7 \quad , n = 6,$

Then

$$\underline{x} = \frac{1 + 0 + 3 + 5 + 2 + 7}{6} = \frac{18}{6} = 3$$
$$x_{+1} = 0 , 1 , 2 , 3 , 5 , 7$$
$$\underline{x}_{+1} = \frac{0}{2^5} + \frac{1}{2^5} + \frac{2}{2^4} + \frac{3}{2^3} + \frac{5}{2^2} + \frac{7}{2^1}$$
$$= 0 + 0.03125 + 0.125 + 0.375 + 1.25 + 3.5 = 5.28125$$
$$x_{-1} = 7 , 5 , 3 , 2 , 1 , 0$$
$$\underline{x}_{+1} = \frac{7}{2^5} + \frac{5}{2^5} + \frac{3}{2^4} + \frac{2}{2^3} + \frac{1}{2^2} + \frac{0}{2^1}$$
$$= 0.21875 + 0.15625 + 0.1875 + 0.25 + 0.25 + 0 = 1.0625$$
$$\underline{x}_0 = \frac{5.28125 + 1.0625}{2} = \frac{6.34375}{2} = 3.171875$$

2. Results and Discussion

The numerical difference between two numbers and the relative difference between two numbers :

Let the two numbers are x_1 , x_2

Let $\{x_1 , x_2\} = \{a, b\}$, such that $a \geq b$, then the numerical difference (ND) and the relative difference (RD) between two numbers x_1 , x_2

$$ND\{x_1, x_2\} = Max\{x_1, x_2\} - Min\{x_1, x_2\} = |x_1 - x_2| = |x_2 - x_1| \; ND\{x_1, x_2\} = a - b RD\{x_1, x_2\} = \frac{Max\{x_1, x_2\} - Min\{x_1, x_2\}}{Max\{x_1, x_2\}} = \frac{ND\{x_1, x_2\}}{Max\{x_1, x_2\}} = \frac{|x_1 - x_2|}{Max\{x_1, x_2\}} = \frac{a-b}{a} = 1 - \frac{b}{a} = 1 - \frac{Min\{x_1, x_2\}}{Max\{x_1, x_2\}}$$

Remark 2-1 :

1. if $Max\{x_1, x_2\} \geq 0$, then $0 \leq RD\{x_1, x_2\} \leq 1$ And if $Max\{x_1, x_2\} \leq 0$, then $-1 \leq RD\{x_1, x_2\} \leq 0$
2. if $x_1 = x_2$, then $Max\{x_1, x_2\} = Max\{x_1, x_2\} = x_1 = x_2$
   $\Rightarrow ND\{x_1, x_2\} = x_1 - x_2 = 0$, $RD\{x_1, x_2\} = 1 - \frac{Min\{x_1, x_2\}}{Max\{x_1, x_2\}} = 1 - \frac{x_1}{x_2} = 1 - 1 = 0$
   For example : $ND\{1, 4\} = 4 - 1 = 3$, $RD\{1, 4\} = \frac{3}{4} = 0.75$

Example 2-2 :

Let $x = 2, 1, 3$

$$\Rightarrow x_{+1} = y = 1, 2, 3 \quad , \quad x_{-1} = z = 3, 2, 1$$

$$\underline{x} = \frac{2+1+3}{3} = \frac{6}{3} = 2$$

$$\underline{x}_{+1} = \frac{1+2+2\times 3}{2^{3-1}} = \frac{9}{4} = 2.25$$

$$\underline{x}_{-1} = \frac{3+2+2\times 1}{4} = \frac{7}{4} = 1.75$$

$$\underline{x}_0 = \frac{\underline{x}_{+1} + \underline{x}_{-1}}{2} = \frac{2.25 + 1.75}{2} = \frac{4}{2} = 2$$

$$ND\{\underline{x}, \underline{x}_0\} = Max\{\underline{x}, \underline{x}_0\} - Min\{\underline{x}, \underline{x}_0\} = |\underline{x} - \underline{x}_0| = |2 - 2| = 0$$

$$RD\{\underline{x}, \underline{x}_0\} = \frac{Max\{\underline{x}, \underline{x}_0\} - Min\{\underline{x}, \underline{x}_0\}}{Max\{\underline{x}, \underline{x}_0\}} = \frac{|\underline{x} - \underline{x}_0|}{Max\{\underline{x}, \underline{x}_0\}} = \frac{ND\{\underline{x}, \underline{x}_0\}}{Max\{\underline{x}, \underline{x}_0\}} = \frac{0}{2} = 0$$

Remark 2-3 :

If $n = 2$, then $\underline{x} = \underline{x}_{+1} = \underline{x}_{-1} = \underline{x}_0 \Rightarrow ND\{\underline{x}, \underline{x}_0\} = RD\{\underline{x}, \underline{x}_0\} = 0$

Theorem 2-4 : If $x = 1, 2, 3, \ldots, n$, then $\underline{x} = \underline{x}_0$

And $ND\{\underline{x}, \underline{x}_0\} = RD\{\underline{x}, \underline{x}_0\} = 0$

Proof :

Let $x = 1, 2, 3, \ldots, n$

$$\Rightarrow \underline{x} = \frac{1+2+3+\cdots+n}{n} = \frac{\left[\frac{n(n+1)}{2}\right]}{n} = \frac{n(n+1)}{2n} = \frac{(n+1)}{2}$$

$$x_{+1} = 1, 2, 3, \ldots, n$$

$$\underline{x}_{+1} = \frac{1+2+2(3)+2^2(4)+\cdots+2^{n-2}n}{2^{n-1}}$$

$$x_{-1} = n, n-1, n-2, \ldots, 1$$

$$\underline{x}_{-1} = \frac{n+(n-1)+2(n-2)+2^2(n-3)+\cdots+2^{n-2}(1)}{2^{n-1}}$$

$$\underline{x}_{+1} + \underline{x}_{-1} = \frac{(n+1)+(n+1)+2(n+1)+2^2(n+1)+\cdots+2^{n-2}(n+1)}{2^{n-1}}$$

$$= (n+1)\frac{1+1+2+2^2+\cdots+2^{n-2}}{2^{n-1}} = (n+1)\frac{1+\frac{1(1-2^{n-1})}{1-2}}{2^{n-1}}$$

$$= (n+1)\frac{1+2^{n-1}-1}{2^{n-1}} = (n+1)\frac{2^{n-1}}{2^{n-1}} = (n+1)$$

$$\underline{x}_0 = \frac{\underline{x}_{+1} + \underline{x}_{-1}}{2} = \frac{n+1}{2} = \underline{x}$$
$$\Longrightarrow \quad ND\{\underline{x}\,, \underline{x}_0\} = RD\{\underline{x}\,, \underline{x}_0\} = 0$$

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