

Article

# Analysis of Survival Times for Cancer Patients using Zero-Truncated Logistic Distribution

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**Abstract:** This research investigates three methods—Maximum Likelihood Estimation (MLE), Trimmed Omission Method (TOM), and Method of Moments (MOM)—for estimating survival time of patients with cancerous tumors using the Log-Logistic Distribution. Background understanding indicates a need for precise survival time estimation to improve patient prognosis and treatment planning. Addressing this gap, the study aims to evaluate these methods using the integrative mean square error (IMSE) criterion. Results demonstrate the superiority of MLE over TOM and MOM in estimating the survival function for the log-logistic distribution of cancer patient data. Additionally, it confirms that the survival function decreases over time, aligning with theoretical expectations. These findings underscore the practical relevance of MLE in clinical applications for predicting patient outcomes.

**Keywords:** log – logistic, Maximum Likelihood, TOM method, MOM method, IMSE, Survival Function.

## 1. Introduction

People are exposed to many healthy problems and diseases, including cancer, which is one of the main causes of death in all the world, including Iraq. As the rate of infection with this disease is constantly increasing in Iraq, especially in recent years, due to wars, the influence of oil industries, and pollution, in addition to the lack of control on this disease compared to other diseases.

Survival analysis is an analysis of time data or the time of occurrence of an event such as a machine stopping working or a patient dying. In this research, the survival time of patients with cancerous tumors in Babil Governorate was studied. In order to obtain estimators with good characteristics. A comparison between these estimators has been studied, in order to choose the best one that represents data of people with cancerous tumors depending on the statistical criterion of the integral mean square error (IMSE). The research deals with the comparison between these methods for estimating the survival time of patients with cancerous tumors according to log-logistic distribution, which are (the Maximum Likelihood method), the Trimmed Omission method, and the Moments method. The study reveals priority of the Maximum Likelihood method.

**Citation:** Shaymaa Mahood Mohammed Analysis of Survival Times for Cancer Patients using Zero-Truncated Logistic Distribution Central Asian Journal of Mathematical Theory and Computer Sciences 2024, 5(3), 106-115.

Received: 21<sup>st</sup> May 2024  
Revised: 21<sup>st</sup> June 2024  
Accepted: 28<sup>th</sup> June 2024  
Published: 4<sup>th</sup> June 2024



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## 2. Materials and Methods

The increase in the incidence of cancerous diseases and their widespread spread, which was the main cause of death for many infected people, especially in Iraq which requires the need to study, analyze, and estimate the survival time of patients with malignant tumors, and thus determine the best estimation method for representing survival time data for patients.

The research aims to find the best method for estimating survival time by comparing three methods for estimating survival time for log- logistic distribution, which are (Maximum Likelihood Estimation Method), (Trimmed Omission Method), and (Moments Method) for patients with malignant tumors using a criterion of the integral mean square error (IMSE).

### Failure Density Function

If  $T$  represents a positive random variable that represents the failure time, then it has a function that measures the probability of the vehicle failing or stopping during the period ( $t < T < t + \Delta t$ )

whatever the value of the change in time ( $\Delta t$ ), it can be expressed mathematically as follows:

$$f_T(t) = \lim_{\Delta t \rightarrow 0} \frac{\Pr(t < T < t + \Delta t)}{\Delta t} ; t \geq 0 \quad (1)$$

It is a single-valued function for each failure time

$$\int_0^{\infty} f_T(t) dt = 1 \quad \text{and} \quad 0 \leq f_T(t) \leq 1 .$$

### Failure Cumulative Function

It is the probability of failure or stopping work for the failure time ( $t$ ) and is expressed mathematically as follows:

$$F_T(t) = \Pr(T < t) = \int_0^t f(u) du ; t \geq 0 \quad (2)$$

It is called the cumulative distribution function (cdf) of failure up to time  $t$ . It is a non-decreasing function for any failure time.

### Survival Function [1-4]

The survival function, or the complementary cumulative distribution function (CDF), is one of the basic concepts that measures the probability of surviving of a person after a limited period of work. That is, it is a person's ability to survive, under specific environmental conditions. It is expressed mathematically as follow:

$$R_T(t) = P(T > t) = 1 - P(T \leq t) = \int_t^{\infty} f_T(u) du = 1 - \int_0^t f_T(u) du = 1 - F_T(t) = \bar{F}(t) \quad (3)$$

This function has some properties, which are that the probability of survival during time ( $t=0$ ) is equal to 1, that is  $R(0) = 1$  that is  $\lim_{t \rightarrow 0} R(t) = 1$  The probability of survival during time ( $t = \infty$ ) is equal to zero, that is  $R(\infty) = 0$  that is  $\lim_{t \rightarrow \infty} R(t) = 0$  meaning that when time increases then, likely the person is died. If  $0 \leq S(t) \leq 1$ ,  $F(t) + S(t) = 1$ , and  $S(t)$  is a continuous decreasing function by the left hand.

### Log-Logistic Distribution [5-8]

It is a continuous probability distribution of a positive random variable. It is used in survival analysis as a parametric model for events whose rate increases initially and subsequently decreases. For example, the rate of death resulting from cancer after diagnosis or treatment. It has also been used in hydrology to model stream flow and

rainfall, in economics as a simple model of the distribution of wealth or income. Also in networking to model data transfer times taking into account both the network and the software.

The logarithmic distribution is probability distribution of a random variable whose logarithm has a logistic distribution. It is similar to shape to the natural logarithmic distribution but has more heavy tails. Unlike natural logarithm, the cumulative distribution function can be written in closed form. The probability density function for a logistic distribution can be formed as follows:

$$f(x; \alpha, \beta) = \frac{\beta e^{-\left(\frac{x}{\alpha}\right)^{\beta-1}}}{\left[1 + e^{-\left(\frac{x}{\alpha}\right)^{\beta}}\right]^2} ; 0 < x < \infty, \mu, \sigma > 0 \quad (4)$$

Since  $t$  is the survival time,  $\alpha$  and  $\beta$  are parameters of the shape of the distribution

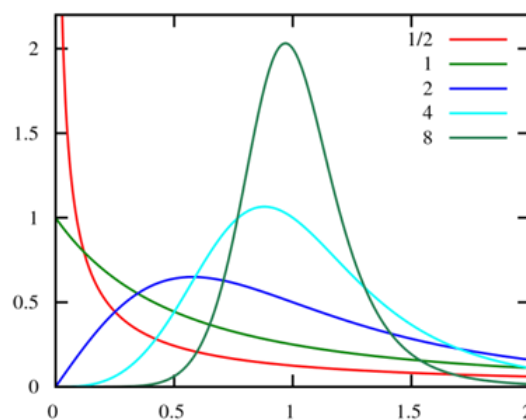


Figure 1. Probability Density Function for the Log-Logistic Distribution

The cumulative distribution function can be written in the following form:

$$F(x; \alpha, \beta) = \frac{1}{1 + e^{-\left(\frac{x}{\alpha}\right)^{\beta}}} ; 0 < x < \infty \quad (5)$$

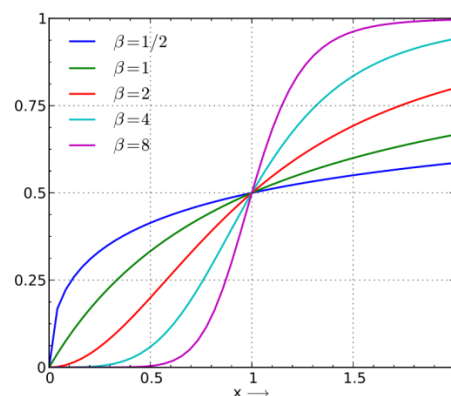


Figure 2. Cumulative Probability Density Function for the Log-Logistic Distribution

The survival function for the distribution is as follows:

$$S(x; \alpha, \beta) = 1 - F(x; \alpha, \beta) = \frac{1}{1 + e^{-\left(\frac{x}{\alpha}\right)^{\beta}}} \quad (6)$$

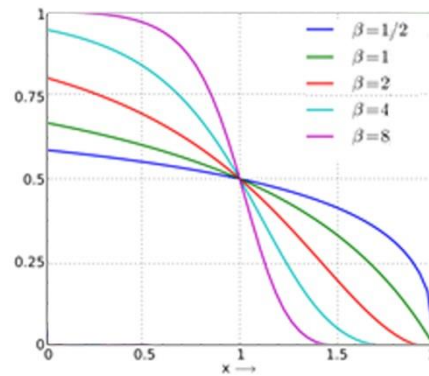


Figure 3. Survival Function for Log- Logistic Distribution.

The central moment in degree K is as follows:

$$\begin{aligned} Ex^k &= \alpha^k B\left(1 - \frac{k}{\beta}; 1 + \frac{k}{\beta}\right) \\ &= \alpha^k \frac{k\pi/\beta}{\sin(k\pi/\beta)} \end{aligned} \quad (7)$$

So B is the missing beta function Therefore, when k=1, we get the average of the distribution (the first central moment)

$$Ex = \alpha \frac{k\pi/\beta}{\sin(k\pi/\beta)} \quad (8)$$

Therefore, when k=2, we obtain the second central moment

$$Ex^2 = \alpha^2 \frac{2\pi/\beta}{\sin(2\pi/\beta)} \quad (9)$$

The variance of the distribution is:

$$var(x) = \alpha^2 \left( \frac{2\pi/\beta}{\sin(2\pi/\beta)} - \frac{\beta^2}{\sin^2(\beta)} \right) \quad (10)$$

The quantitative distribution function is:

$$Q(x) = \alpha \left( \frac{u}{1-u} \right)^{\frac{1}{\beta}} \quad (11)$$

### 1. Maximum Likelihood for Log-Logistic Distribution [9-11]

Let us have  $x_1, x_2, \dots, x_n$  A random sample has a log- logistic distribution, as in equation (4). The possibility function for the distribution is written as follows:

$$\begin{aligned} L &= \prod_{i=1}^n f(x; \alpha, \beta) \\ &= \prod_{i=1}^n \frac{\frac{\beta}{\alpha} e^{-(\frac{x}{\alpha})^{\beta-1}}}{\left[1 + e^{-(\frac{x}{\alpha})^{\beta}}\right]^2} \\ &= \left(\frac{\beta}{\alpha}\right)^n \prod_{i=1}^n \frac{e^{-(\frac{x}{\alpha})^{\beta-1}}}{\left[1 + e^{-(\frac{x}{\alpha})^{\beta}}\right]^2} \end{aligned} \quad (12)$$

Taking the natural logarithm of equation (12) we get:

$$l = \ln(L) = n \ln(\beta) - n \ln(\alpha) + \beta \sum_{i=1}^n \ln(x_i) - \beta \sum_{i=1}^n \ln(\alpha) - \sum_{i=1}^n \ln(x_i) + \sum_{i=1}^n \ln(\alpha) - 2 \sum_{i=1}^n \ln\left(1 + \left(\frac{x_i}{\alpha}\right)^{\beta}\right) \quad (13)$$

By taking the first derivative of equation (13) and setting it equal to zero for the parameters to be estimated, we obtain the following:

$$\begin{aligned}\frac{\partial l}{\partial \hat{\alpha}} &= \frac{2\hat{\beta}}{\hat{\alpha}} \sum_{i=1}^n \frac{\left(\frac{x_i}{\hat{\alpha}}\right)^{\hat{\beta}}}{1 + e^{-\left(\frac{x_i}{\hat{\alpha}}\right)^{\hat{\beta}}}} - \frac{n\hat{\beta}}{\hat{\alpha}} = 0 \\ &= 2 \sum_{i=1}^n \frac{\left(\frac{x_i}{\hat{\alpha}}\right)^{\hat{\beta}}}{1 + e^{-\left(\frac{x_i}{\hat{\alpha}}\right)^{\hat{\beta}}}} - n = 0\end{aligned}\quad (14)$$

$$\begin{aligned}\frac{\partial l}{\partial \hat{\beta}} &= \frac{n}{\hat{\beta}} + \sum_{i=1}^n \ln\left(\frac{x_i}{\hat{\alpha}}\right) - 2 \sum_{i=1}^n \frac{\left(\frac{x_i}{\hat{\alpha}}\right)^2 \ln\left(\frac{x_i}{\hat{\alpha}}\right)}{1 + e^{-\left(\frac{x_i}{\hat{\alpha}}\right)^{\hat{\beta}}}} = 0 \\ &= \frac{n}{\hat{\beta}} + \sum_{i=1}^n \ln\left(\frac{x_i}{\hat{\alpha}}\right) - 2 \sum_{i=1}^n \frac{\ln\left(\frac{x_i}{\hat{\alpha}}\right)}{1 + e^{-\left(\frac{x_i}{\hat{\alpha}}\right)^{\hat{\beta}}}} = 0\end{aligned}\quad (15)$$

Equations (14) and (15) are solved by numerical iterative methods using the fsolve function in the Matlab program.

## 2. Moments Method (MOM) [1], [4], [12].

The first moment of distribution is:

$$M_1 = E(x) = \alpha \frac{k\pi/\beta}{\sin(k\pi/\beta)}$$

By equating the first sample moment with the first population moment, we obtain the following equation, from which we find the estimate of the parameter  $\alpha$  as follows:

$$\hat{M}_1 = \bar{X}$$

$$\alpha \frac{k\pi/\beta}{\sin(k\pi/\beta)} = \bar{X}\quad (16)$$

By equating the second sample moment with the second population moment, we obtain the following equation, from which we find the estimate of the parameter  $\beta$  as follows:

$$\begin{aligned}M_2 &= Var(x) + M_1^2 \\ &= \alpha^2 \left( \frac{2\beta}{\sin(2\beta)} - \frac{\beta^2}{\sin^2(\beta)} \right) + \left( \alpha \frac{k\pi/\beta}{\sin(k\pi/\beta)} \right)^2 \\ M_2 &= \frac{\sum_{i=1}^n x_i^2}{n} \\ \hat{M}_2 = M_2 &= \alpha^2 \left( \frac{2\beta}{\sin(2\beta)} - \frac{\beta^2}{\sin^2(\beta)} \right) + \left( \alpha \frac{k\pi/\beta}{\sin(k\pi/\beta)} \right)^2 = \frac{\sum_{i=1}^n x_i^2}{n}\end{aligned}\quad (17)$$

Equations (16) and (17) are solved by numerical iterative methods using the fsolve function in the Matlab program.

## 3. Trimmed Omission method for Log-Logistic Distribution [13-16]

This method was introduced by (Jubron AA.K) in (2016) and depends on making the error between any two values in the sample field  $x_{ik}$  and  $x_{ik+1}$  to the smallest possible, relying on the probability density function of the distribution as follows:

The probability density function (pdf) of the distribution is as in equation (4) and for any two values of  $x_i$  and  $x_{i+1}$   $x_i$  be such that

$$1 \leq x_i \leq x_{i+1} \leq n$$

The probability density function for each observation is written as follows:

$$x_i \quad \tilde{f}(x_i) = \frac{\frac{\beta}{\alpha} e^{-\left(\frac{x_i}{\alpha}\right)^{\beta-1}}}{\left[1 + e^{-\left(\frac{x_i}{\alpha}\right)^{\beta}}\right]^2} \quad (18)$$

$$x_{i+1} \quad \tilde{f}(x_{i+1}) = \frac{\frac{\beta}{\alpha} e^{-\left(\frac{x_{i+1}}{\alpha}\right)^{\beta-1}}}{\left[1 + e^{-\left(\frac{x_{i+1}}{\alpha}\right)^{\beta}}\right]^2} \quad (19)$$

To find Tom's estimator for the parameter  $\alpha$ , we divide equation (18) by equation (19). Then :

$$\frac{x_i}{x_{i+1}} = \frac{e^{-\left(\frac{x_i}{\alpha}\right)^{\beta-1}} \left[1 + e^{-\left(\frac{x_{i+1}}{\alpha}\right)^{\beta}}\right]^2}{e^{-\left(\frac{x_{i+1}}{\alpha}\right)^{\beta-1}} \left[1 + e^{-\left(\frac{x_i}{\alpha}\right)^{\beta}}\right]^2} \quad (20)$$

Then we take the natural logarithm of both sides as follows:

$$\ln\left(\frac{x_i}{x_{i+1}}\right) = \ln\left(\frac{e^{-\left(\frac{x_i}{\alpha}\right)^{\beta-1}} \left[1 + e^{-\left(\frac{x_{i+1}}{\alpha}\right)^{\beta}}\right]^2}{e^{-\left(\frac{x_{i+1}}{\alpha}\right)^{\beta-1}} \left[1 + e^{-\left(\frac{x_i}{\alpha}\right)^{\beta}}\right]^2}\right) \quad (21)$$

The value of  $\alpha$  at iteration k is:

$$\alpha^k = \frac{\ln\left(\frac{e^{-\left(\frac{x_i}{\alpha}\right)^{\beta-1}} \left[1 + e^{-\left(\frac{x_{i+1}}{\alpha}\right)^{\beta}}\right]^2}{e^{-\left(\frac{x_{i+1}}{\alpha}\right)^{\beta-1}} \left[1 + e^{-\left(\frac{x_i}{\alpha}\right)^{\beta}}\right]^2}\right)}{\ln x_{ik} - \ln x_{ik+1}} \quad (22)$$

The TOM estimator for the parameter  $\alpha$  is the estimator that makes the sum of the squares of the difference between the cumulative function  $\alpha^k$  at in the K iteration of the parameter  $\alpha$  and a certain value of x at k as small as possible, that is:

$$\hat{\alpha}_{Tom}^k = \min_{1 \leq k < n-1} \sum_{k=1}^{n-1} [F(x_{ik}, \alpha^k) - x_{ik}]^2 \quad (23)$$

$$k = 1, 2, 3, \dots, n-1 \quad \text{and} \quad i = 1, 2, 3, \dots, n$$

In the same way, we find the Tom estimator for the parameter  $\beta$ . The TOM estimator for the parameter  $\beta$  is the estimator that makes the sum of the squares of the difference between the cumulative function at  $\beta^k$  in the K iteration of the parameter  $\beta$  and a certain value of x at k as least as possible, that is:

$$\hat{\beta}_{Tom}^k = \min_{1 \leq k < n-1} \sum_{k=1}^{n-1} [F(x_{ik}, \beta^k) - x_{ik}]^2 \quad (24)$$

$$k = 1, 2, 3, \dots, n-1 \quad \text{and} \quad i = 1, 2, 3, \dots, n$$

We note that equation (23) and equation (24) are not solved using the ordinary analytical methods. Therefore, one of the methods is used, the numerical iterative method, which is the fsolve function in the MATLAB program, and the estimator  $\hat{\alpha}_{Tom}^k$  and the estimator  $\hat{\beta}_{Tom}^k$  are found. Tom) and then substitute the estimators  $(\hat{\alpha}_{Tom}^k, \hat{\beta}_{Tom}^k)$  into the survival function of the agency distribution:

$$\hat{S}_{tom}(x) = \frac{1}{1 + e^{-\left(\frac{x}{\hat{\alpha}^k_{Tom}\right)^{\hat{\beta}^k_{Tom}}}} \quad (25)$$

### Applied Part

We used real data represent survival times for people with cancerous tumors, which were taken from the Babylon Center for Oncology Treatment for the years from 2020 to 2023, with (100) observations taken randomly by month and converted to daily observations by dividing each observation by the number of days in the month to obtain continuous data. Studying the data by using the Chi-square test, giving the value of Sig. = 0.0000, which is less than the significance level of 0.01, and therefore the data follows a logarithmic logistic distribution.

Table 1. Values of the True Survival Function Estimated by Estimation Methods for the Log-Distribution.

x	S_real	S_mle	S_mom	S_tom
0.06	0.97402	0.97576	0.88787	0.96994
0.21	0.91213	0.91778	0.66454	0.89900
0.21	0.91126	0.91696	0.66177	0.89800
0.23	0.90082	0.90715	0.62956	0.88614
0.25	0.89454	0.90124	0.61081	0.87900
0.25	0.89406	0.90079	0.60941	0.87846
0.27	0.88375	0.89108	0.57976	0.86678
0.34	0.85468	0.86365	0.50270	0.83399
0.35	0.85014	0.85937	0.49150	0.82890
0.37	0.84349	0.85308	0.47546	0.82143
0.37	0.84283	0.85245	0.47388	0.82068
0.41	0.82437	0.83497	0.43178	0.80002
0.42	0.82011	0.83094	0.42253	0.79527
0.46	0.80370	0.81537	0.38841	0.77700
0.48	0.79809	0.81005	0.37729	0.77077
0.48	0.79809	0.81004	0.37729	0.77077
0.49	0.79363	0.80580	0.36863	0.76582
0.50	0.78961	0.80198	0.36097	0.76136
0.52	0.78068	0.79348	0.34442	0.75147
0.56	0.76377	0.77738	0.31482	0.73281
0.57	0.75924	0.77306	0.30725	0.72782
0.58	0.75550	0.76949	0.30112	0.72371
0.60	0.74866	0.76297	0.29016	0.71620
0.61	0.74292	0.75749	0.28123	0.70991
0.70	0.70887	0.72490	0.23268	0.67273
0.71	0.70366	0.71991	0.22589	0.66707
0.73	0.69802	0.71450	0.21872	0.66095
0.74	0.69290	0.70957	0.21237	0.65539
0.77	0.68302	0.70008	0.20054	0.64471
0.78	0.67780	0.69507	0.19452	0.63908
0.78	0.67769	0.69496	0.19439	0.63895
0.81	0.66565	0.68338	0.18104	0.62599
0.82	0.66522	0.68296	0.18057	0.62552
0.85	0.65160	0.66983	0.16640	0.61090

0.89	0.63741	0.65613	0.15258	0.59571
0.96	0.61396	0.63344	0.13176	0.57074
0.98	0.60743	0.62711	0.12638	0.56381
0.98	0.60470	0.62447	0.12419	0.56092
0.99	0.60374	0.62354	0.12342	0.55990
1.02	0.59226	0.61240	0.11456	0.54777
1.02	0.59163	0.61179	0.11408	0.54710
1.03	0.58841	0.60866	0.11169	0.54370
1.08	0.57311	0.59378	0.10088	0.52760
1.08	0.57173	0.59244	0.09995	0.52616
1.15	0.54754	0.56885	0.08466	0.50085
1.15	0.54727	0.56859	0.08450	0.50057
1.22	0.52589	0.54768	0.07258	0.47835
1.23	0.52168	0.54356	0.07039	0.47400
1.24	0.52145	0.54333	0.07027	0.47376
1.24	0.51855	0.54049	0.06880	0.47076
1.25	0.51762	0.53958	0.06833	0.46980
1.34	0.49043	0.51288	0.05571	0.44182
1.36	0.48496	0.50750	0.05341	0.43622
1.38	0.47772	0.50037	0.05047	0.42881
1.39	0.47515	0.49783	0.04946	0.42619
1.39	0.47326	0.49597	0.04872	0.42426
1.45	0.45712	0.48004	0.04279	0.40786
1.47	0.45176	0.47474	0.04094	0.40242
1.51	0.44018	0.46328	0.03717	0.39073
1.62	0.41112	0.43443	0.02885	0.36157
1.62	0.41104	0.43434	0.02883	0.36149
1.64	0.40600	0.42933	0.02755	0.35647
1.69	0.39385	0.41722	0.02463	0.34439
1.72	0.38486	0.40824	0.02263	0.33548
1.84	0.35645	0.37978	0.01710	0.30755
1.95	0.33371	0.35690	0.01345	0.28542
1.97	0.32751	0.35065	0.01257	0.27942
1.98	0.32683	0.34997	0.01248	0.27877
1.98	0.32664	0.34976	0.01245	0.27858
2.06	0.30916	0.33209	0.01021	0.26177
2.08	0.30449	0.32736	0.00967	0.25731
2.28	0.26746	0.28966	0.00608	0.22221
2.30	0.26323	0.28534	0.00574	0.21825
2.31	0.26152	0.28359	0.00561	0.21664
2.44	0.24064	0.26217	0.00418	0.19721
2.56	0.22063	0.24154	0.00307	0.17881
2.71	0.19978	0.21992	0.00217	0.15986
2.72	0.19751	0.21756	0.00209	0.15782
2.73	0.19714	0.21717	0.00207	0.15748
2.95	0.16851	0.18724	0.00120	0.13195
3.09	0.15267	0.17057	0.00085	0.11807
3.18	0.14371	0.16108	0.00069	0.11030
3.36	0.12641	0.14268	0.00045	0.09548
3.44	0.11934	0.13512	0.00037	0.08949

<b>3.58</b>	<b>0.10778</b>	<b>0.12270</b>	<b>0.00026</b>	<b>0.07981</b>
<b>3.58</b>	<b>0.10726</b>	<b>0.12214</b>	<b>0.00025</b>	<b>0.07937</b>
<b>3.62</b>	<b>0.10472</b>	<b>0.11940</b>	<b>0.00023</b>	<b>0.07726</b>
<b>3.70</b>	<b>0.09843</b>	<b>0.11260</b>	<b>0.00019</b>	<b>0.07207</b>
<b>3.77</b>	<b>0.09392</b>	<b>0.10772</b>	<b>0.00016</b>	<b>0.06838</b>
<b>3.95</b>	<b>0.08221</b>	<b>0.09497</b>	<b>0.00010</b>	<b>0.05888</b>
<b>4.03</b>	<b>0.07754</b>	<b>0.08984</b>	<b>0.00008</b>	<b>0.05514</b>
<b>4.18</b>	<b>0.06926</b>	<b>0.08073</b>	<b>0.00006</b>	<b>0.04857</b>
<b>4.25</b>	<b>0.06578</b>	<b>0.07689</b>	<b>0.00005</b>	<b>0.04585</b>
<b>4.56</b>	<b>0.05229</b>	<b>0.06186</b>	<b>0.00002</b>	<b>0.03544</b>
<b>4.57</b>	<b>0.05173</b>	<b>0.06123</b>	<b>0.00002</b>	<b>0.03502</b>
<b>4.78</b>	<b>0.04427</b>	<b>0.05283</b>	<b>0.00001</b>	<b>0.02941</b>
<b>4.86</b>	<b>0.04164</b>	<b>0.04986</b>	<b>0.00001</b>	<b>0.02746</b>
<b>5.35</b>	<b>0.02859</b>	<b>0.03491</b>	<b>0.00000</b>	<b>0.01803</b>
<b>7.41</b>	<b>0.00573</b>	<b>0.00759</b>	<b>0.00000</b>	<b>0.00299</b>
<b>7.45</b>	<b>0.00555</b>	<b>0.00736</b>	<b>0.00000</b>	<b>0.00289</b>
<b>IMSE</b>		<b>0.00026</b>	<b>0.12888</b>	<b>0.00031</b>
<b>Rank</b>		<b>1</b>	<b>3</b>	<b>2</b>

### 3. Results and Discussion

From Table (1) and Figure (4) we notice that the estimated survival function for the log-logistic distribution is decreasing with increasing time, that is, it is inversely proportional to time. That is, it is inversely proportional to time. As the results of Table (1) showed, the superiority of the maximum likelihood estimation method (MLE) over both the method of moments (MOM) and the method of omitting the range (TOM) in representing the data of patients with cancerous tumors because it gave the mean square error standard(IMSE) (IMSE), which reached (0.00026), While the term elimination method (TOM) outperformed the moments method (MOM), as the integrative mean square error standard(IMSE) reached (0.00031).

### 4. Conclusion

The research demonstrates that the survival function decreases with time, consistent with theoretical expectations. Additionally, it establishes the superiority of the Maximum Likelihood Estimation (MLE) method over the Methods of Moments (MOM) and Trimmed Omission Method (TOM) in estimating the survival function for the log-logistic distribution of data from patients with cancerous tumors. Furthermore, the study finds that the Trimmed Omission Method is superior to the Methods of Moments in estimating the survival function for the log-logistic distribution.

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