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Article

Exploring the Wonders of the Viessman-Gray Manifold

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Abstract: In this study, we investigate the geometric properties of the Generalized Viessman-Gray manifold, focusing on its conharmonic curvature tensor and its relationship with flat conharmonic curvature tensors. Leveraging the fleetness property of the GT-manifold, we utilize Weyl's tensor to derive components of the conformal GT-manifold and establish its classification as a Nearly Kähler manifold due to its zero scalar curvature. Additionally, we uncover the connection between the GT-manifold and conharmonic Para-Kähler manifolds. Through rigorous analysis and proof, we demonstrate that a GT-manifold with flat conharmonic tensors is Nearly Kähler and possesses zero scalar curvature, establishing its classification as a conharmonic Para-Kähler manifold. Our findings contribute to the understanding of these manifold structures and their geometric properties, paving the way for further exploration in related fields.

Keywords: Almost Hermitian Manifold, Conharmonic Curvature tensor, Generalized Viessman - Gray manifold, Locally conformal Kahler, Differential geometry.

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1. Introduction

It is considered the Hermitian manifold one of the most important subjects in Geometry differential, so the divided into a number of components in an effort to precisely identify its characteristics to Hermitian manifold's classes being divided based on distinct properties. Many academics investigated the nearly Hermitian manifold and discovered that it has interesting geometrical qualities. A Russian researcher named Kirichenko is among them, he discovered the adjoined G-structure space while studying the nearly Hermitian manifold, which is not dependent on a manifold but on a subprinciple of all complex frames' aggregate fiber bundle.[1] Ali. Shihab & Ali Khalif studied the W-Projective curvature of nearly Kahler manifold [2]. Maath and Ali studied the geometry of conhormonic and calculated components of conharmonic curvature tensor of the Locally Conformal Kahler manifold [3].

Several academics have examined the geometric features of certain types of manifolds using curvature tensors on virtually Hermitian manifolds, where Kirichenko and Rustanov conducted a study on the geometric properties of conhormonic curvature in nearly Hermitian manifolds [4]. Elham and. Shihab studied the Generalized Conharmonic Curvature Tensor of the Locally Conformal Kahler Manifold and shows that this tensor possesses the classic symmetry properties of the Riemannian tensor. And find relationships between the components of the tensor in this manifold [5]. Abdulhadi & Ali studied the geometry properties of generalized conharmonic tensor of the Viessman- Grey manifold [6]. The researcher uses the flatness property Generalized Vaisman-Gray manifold (GT-manifold) to find the components of conformal Generalized

Viessman-gray manifold (GT-manifold) by the Weyl`s tensor and proved that the manifold is Nearly Kahler manifold (NK-manifold), and show that the manifold has a zero scalar curvature. Finally, the relation between the Generalized Viessman -gray manifold (GT-manifold) and conharmonic Para-Kahler manifold has been found.

2. Materials and Methods

Denote the smooth vector field module on M as X(ᴍ). C^{∞} (ᴍ) the group of smooth activities of ᴍ. Almost Hermitian (AH-manifold) is a group { ᴍ, ᴊ, g=⟨.,.⟩}, ᴍ is 2n- A smooth manifold with more than one dimension, and an endomorphism of the Riemannian R. so $\langle JZ, JW\rangle = \langle Z, \mathcal{W}\rangle; Z, W \in Z(\text{ᴍ})$

The fundamentals $\{e_1 \ldots\ldots, e_n \ldots Je_1 \ldots . Je_n\}$ is the follow $\{i_1, \ldots i_n, \ldots \bar{i}_1, \ldots, \bar{i}_n\}$. There are signs i, j, k as well as l of the vicinity $1, \ldots, 2n$ and h. The interval 1,2,...,n. We will use these notations $\{i_{\hat{1}} = \bar{i}_1, \ldots . i_{\hat{n}} = \bar{i}_n\}$ where $\hat{a} = a + n.$ form might be utilized in order to compose an a-frame $\{p, i_1 \ldots, i_n \ldots ., i_{\hat{\hat{1}}}, \ldots i_{\hat{n}}\}$. The complex structures' component matrices J and g in the adjoined G-structure are several types of space:

$$(\langle JX, JY\rangle J_j^i) = \begin{pmatrix} \sqrt{-1}\, I_n & 0 \\ 0 & -\sqrt{-1}\, I_n \end{pmatrix}, \ (g_j^i) = \begin{pmatrix} 0 & I_n \\ I_n & 0 \end{pmatrix}.$$

Definition 2.1 [8]

Using Banner's classification of almost hermit, the group of Viessman meets the following criteria: $B^{abc} = -B^{bac}, B_c^{ab} = \alpha^{[a}\delta_c^{b]}$.

Definition.2.2 [9]

A tensor of Riemann curvature R. given a smooth manifold, the number ᴍ is four-covariant.

tenser R: GP(ᴍ)× GP(ᴍ)× GP(ᴍ)× GP(ᴍ)×→R, what it means:

δ (α, β, γ, ε)] = (δ (γ, ε), β, α)

It given the following criteria:

a) δ (α, β, γ, ε) = δ (β, α, γ, ε);

b) δ (α, β, γ, ε) = δ (α, β, ε, γ);

c) δ (α, β, γ ', ε) = δ (γ, ε, α, β);

d) δ (α, β, γ, ε) + δ (α, γ, ε, β) + δ (α, ε, β, γ) =0;

Theorem 2.1 [7]

The following forms can be found in the collections on Viessman-grey structure equations:

i) $d\omega^a = \omega_b^a \wedge \omega^b + B_c^{ab}\omega^c \wedge \omega_b + B^{abc}\omega_b \wedge \omega_c$;

ii) $d\omega_a = -\omega_a^b \wedge \omega_b + B_{ab}^c \omega_c \wedge \omega^b + \omega_{abc}\omega^b \wedge \omega_c$;

iii) $d\omega_b^a \omega_c^a \wedge \omega_b^c + \left(2B^{adh}B_{hbc} + A_{bc}^{ad}\right)\omega^c \wedge \omega_d + \left(B^{ah}{}_{[c}B_{d]bh} + A_{bcd}^a\right)\omega^c \wedge \omega^d + \left(B_{bh}{}^{[c}B^{d]ah} + A_b^{acd}\right)\omega_c \wedge \omega_{d;}$

Where ω^i are the mixed form components and ω_j^i Riemannian connection components of metric g.

Definition 2.3 [9]

A type (2,0) tensor that is described as $r(GT)_{ij} = (GT)_{ijk}^k$ is named a Ricci tensor generalized

Theorem 2.2 [6]

The Viessman gray components of the generalized Riemann curvature there are in the adjoined G-structure provided in formats:

1. $GT_{\hat{a}b\hat{c}d} = \left\{-A^{ac}_{bd} + B^{ah}{}_{b}B_{hd}{}^{c} + \frac{1}{2}\alpha^{[a}_{[b}\delta^{c]}_{d]}\right\}$
2. $GT_{\hat{a}bc\hat{d}} = \left\{A^{ad}_{bc} - B^{ah}{}_{c}B_{hb}{}^{d} + \frac{1}{2}\alpha^{[a}_{[b}\delta^{d]}_{c]}\right\}$.

3. Results

Definition 3.1

It has been discovered the relation between the Generalized Viessman-gray manifold (GT-manifold) and the conharmonic Para_Kohler manifold.

Detention 3.2 [10].

The component of a Viessman manifold is referred to as a conharmonic Para Kahler manifold if $GT_{\hat{a}\hat{b}cd}$ of conharmonic curvature tensor T is equal to zero.

Definition 3.3 [9].

In order to define the A scalar tensor K of an AH-manifold, the Rici tenser is contracted & give $r = g^{ij}r_{ij}$.

Definition 3.4 [9].

As for as the Riemannian space, the conformal curvature tensor or Weyl's tensor $W = W^{i}_{jkl}$ of type (3,1) is defined by the form:

$$W_{ijkl} = R_{ijkl} + \frac{1}{m-1}\left(r_{ik}g_{jl} + r_{jl}g_{ik} - r_{il}g_{jk} - r_{jk}g_{il}\right) + \frac{K(g_{il}g_{jk} - g_{ik}g_{jl})}{(m-2)(m-1)} \quad (3.1)$$

(3.1)

Where R_{ijkl} are the Riemannian curvature tensor components, $r_{il}$are the Ricci tensor's constituent parts., $g_{jk}$are components of the Riemannian metric g and K is the scalar curvature. This tensor is unaffected by conformal transformation metric by as follow:

$$W^{*}{}_{ijkl} = R_{ijkl} + \frac{1}{2(n-1)}\left(r_{ik}g_{jl} + r_{jl}g_{ik} - r_{il}g_{jk} - r_{jk}g_{il}\right) + \frac{K(g_{il}g_{jk} - g_{ik}g_{jl})}{2(n-1)(2n-1)}$$

(3.2)

This tensor has similar properties to those of the Riemannian curvature tensor.

Theorem 3.1

The components of the conformal of the adjoint G-structure of generalized Viessman by the Weyl`s tensor are given by the following form:

Proof:

1. For $i = a, j = b, K = c$, and $\ell = d$,

The equation (3.2) becomes

$$W^{*}{}_{abcd} = R_{abcd} + \frac{1}{2(n-1)}(r_{ac}g_{bd} + r_{bd}g_{ac} - r_{ad}g_{bc} - r_{bc}g_{ad}) + \frac{K(g_{bc}g_{ad} - g_{bd}g_{ac})}{2(n-1)(2n-1)}$$

According to equation (2.1), we get that

$$W^{*}{}_{abcd} = R_{abcd} = 0$$

2. For $i = \hat{a}, j = b, k = c,$ & $l = d$, we have

$$W^{*}{}_{\hat{a}bcd} = R_{\hat{a}bcd} + \frac{1}{2(n-1)}(r_{\hat{a}c}g_{bd} + r_{bd}g_{\hat{a}c} - r_{\hat{a}d}g_{bc} - r_{bc}g_{\hat{a}d}) + \frac{K(g_{bc}g_{\hat{a}d} - g_{bd}g_{\hat{a}c})}{2(n-1)(2n-1)}$$

According to equation (2.1), we get that

$W^*{}_{\hat{a}bcd} = 0 + \frac{1}{2(n-1)}(0 + r_{bd}\delta_c^a - 0 - r_{bc}\delta_d^a) + 0$

$W^*{}_{\hat{a}bcd} = \frac{1}{2(n-1)}(r_{bd}\delta_c^a - r_{bc}\delta_d^a)$

3. For $i = a, j = \hat{b},\ k = c,\ \&\ l = d$, we get:

$W^*{}_{a\hat{b}cd} = R_{a\hat{b}cd} + \frac{1}{2(n-1)}(r_{ac}g_{\hat{b}d} + r_{\hat{b}d}g_{ac} - r_{ad}g_{\hat{b}c} - r_{\hat{b}c}g_{ad}) + \frac{K(g_{\hat{b}c}g_{ad} - g_{\hat{b}d}g_{ac})}{2(n-1)(2n-1)}$

According to equation (2.1), we get that

$W^*{}_{a\hat{b}cd} = 0 + \frac{1}{2(n-1)}\left(r_{ac}\delta_d^b + 0 - r_{ad}\delta_c^b - 0\right) + 0$

$W^*{}_{a\hat{b}cd} = \frac{1}{2(n-1)}\left(r_{ac}\delta_d^b - r_{ad}\delta_c^b\right)$

4. For $i = a, j = b,\ \ k = \hat{c},$ and $l = d$, we have

$W^*{}_{ab\hat{c}d} =$
$R_{ab\hat{c}d} + \frac{1}{2(n-1)}(r_{a\hat{c}}g_{bd} + r_{bd}g_{a\hat{c}} - r_{ad}g_{b\hat{c}} - r_{b\hat{c}}g_{ad}) + \frac{K(g_{b\hat{c}}g_{ad} - g_{bd}g_{a\hat{c}})}{2(n-1)(2n-1)}$

According to equation (2.1), we get that

$W^*{}_{ab\hat{c}d} = 0 + \frac{1}{2(n-1)}(0 + r_{bd}\delta_a^c - r_{ad}\delta_b^c - 0) + 0$

$W^*{}_{ab\hat{c}d} = \frac{1}{2(n-1)}(r_{bd}\delta_a^c - r_{ad}\delta_c^b)$

5. For $i = a\ .\ j = b,\ k = c,\ \&\ l = \hat{d}$, we obtain

$W^*{}_{ab\hat{c}d} =$
$R_{ab\hat{c}d} + \frac{1}{2(n-1)}(r_{a\hat{c}}g_{bd} + r_{bd}g_{a\hat{c}} - r_{ad}g_{b\hat{c}} - r_{b\hat{c}}g_{ad}) + \frac{K(g_{b\hat{c}}g_{ad} - g_{bd}g_{a\hat{c}})}{2(n-1)(2n-1)}$

According to equation (2.1), we get that

$W^*{}_{abc\hat{d}} = 0 + \frac{1}{2(n-1)}\left(r_{ac}\delta_b^d + 0 - 0 - r_{bc}\delta_a^d\right) + 0$

$W^*{}_{abc\hat{d}} = \frac{1}{2(n-1)}\left(r_{ac}\delta_b^d - r_{bc}\delta_a^d\right)$

6. For $i = \hat{a}, j = \hat{b}\ ,\ k = c,\ \&\ l = d$, we obtain

$W^*{}_{\hat{a}\hat{b}cd} = R_{\hat{a}\hat{b}cd} + \frac{1}{2(n-1)}(r_{\hat{a}c}g_{\hat{b}d} + r_{\hat{b}d}g_{\hat{a}c} - r_{\hat{a}d}g_{\hat{b}c} - r_{\hat{b}c}g_{\hat{a}d}) + \frac{K(g_{\hat{b}c}g_{\hat{a}d} - g_{\hat{b}d}g_{\hat{a}c})}{2(n-1)(2n-1)}$

According to equation (2.1), we get that

$W^*{}_{\hat{a}\hat{b}cd} = 0 + \frac{1}{2(n-1)}\left(r_c^a\delta_d^b + r_d^b\delta_c^a - r_d^a\delta_c^b - r_c^b\delta_d^a\right) + \frac{K}{2(n-1)(2n-1)}(\delta_c^b\delta_d^a - \delta_d^b\delta_c^a)$

$W^*{}_{\hat{a}\hat{b}cd} = \frac{1}{4(n-1)}\left(r_{[c}^{[a}\delta_{d]}^{b]}\right) + \frac{K}{2(n-1)(2n-1)}(\delta_{cd}^{ab})$

7. For $i = \hat{a}, j = b, \kappa = \check{c}\ \ \&\ l = d$, we have

$W^*{}_{\hat{a}b\hat{c}d} =$
$R_{\hat{a}b\hat{c}d} + \frac{1}{2(n-1)}(r_{\hat{a}\hat{c}}g_{bd} + r_{bd}g_{\hat{a}\hat{c}} - r_{\hat{a}d}g_{b\hat{c}} - r_{b\hat{c}}g_{\hat{a}d})\frac{K(g_{b\hat{c}}g_{\hat{a}d} - g_{bd}g_{\hat{a}\hat{c}})}{2(n-1)(2n-1)}$

By theorem (2.2), we get that

$$W_{\hat{a}b\hat{c}d} = -A^{ac}_{bd} + B^{ah}{}_{b}B_{hd}{}^{c} + \frac{1}{2}\alpha^{[a}_{[b}\delta^{c]}_{d]} + \frac{1}{2(n-1)}\left(-r^{a}_{d}\delta^{c}_{b} - r^{c}_{b}\delta^{a}_{d}\right) + \frac{K}{2(n-1)(2n-1)}\left(\delta^{c}_{b}\delta^{a}_{d}\right)$$

7. For $\underset{\sim}{i} = \breve{a}, j = b, \kappa = c, \&\ l = \breve{d}$, we have

$$W^{*}{}_{\hat{a}bc\hat{d}} = R_{\hat{a}bc\hat{d}} + \frac{1}{2(\eta-1)}\left(r_{\hat{a}c}g_{b\hat{d}} + r_{b\hat{d}}g_{\hat{a}c} - r_{\hat{a}\hat{d}}g_{bc} - r_{bc}g_{\hat{a}\hat{d}}\right) + \frac{K\left(g_{bc}g_{\hat{a}\hat{d}} - g_{b\hat{d}}g_{\hat{a}c}\right)}{2(n-1)(2n-1)}$$

By theorem (2.2), we get that

$$W^{*}{}_{\breve{a}bc\breve{d}} = A^{ad}_{bc} - B^{ah}{}_{c}B_{hb}{}^{d} + \frac{1}{2}\alpha^{[a}_{[b}\delta^{d]}_{c]} + \frac{1}{2(\eta-1)}\left(-r^{a}_{c}\delta^{d}_{b} - r^{d}_{b}\delta^{a}_{c}\right) + \frac{K}{2(n-1)(2n-1)}\left(\delta^{d}_{b}\delta^{a}_{c}\right).$$

Theorem 3.2

Suppose M be a generalized Viessman -gray manifold (GT-manifold) having a flat Conharmonic, then M is Nearly Kohler.

Proof:

Let M is generalized Viessman_manifold (GT-manifold) having a flat conharmonic tenser By definition (3.1) & theorem (3.1) we obtain

$$A^{ad}_{bc} - B^{ah}{}_{c}B_{hb}{}^{d} + \frac{1}{2}\alpha^{[a}_{[b}\delta^{d]}_{c]} + \frac{1}{2(n-1)}\left(-r^{a}_{c}\delta^{d}_{b} - r^{d}_{b}\delta^{a}_{c}\right) + \frac{K}{2(n-1)(2n-1)}\delta^{d}_{b}\delta^{a}_{c} = 0 \quad (3.1)$$

Symmetrizing and antisymmetrizing equation (3.1) according to the index (a , d) we deduce

$$-B^{ah}{}_{c}B_{hb}{}^{d} = 0 \quad (3.2)$$

A contract-based (3.2) by index (d ¸c),& (a ¸b) we have:

$$B^{ah}{}_{d}B_{ha}{}^{d} = 0 \Rightarrow B^{ah}{}_{d}\overline{B}^{ah}{}_{d} = 0 \Leftrightarrow \sum_{a,h,d}\left|B^{ah}{}_{d}\right|^{2} = 0 \Leftrightarrow B^{ah}{}_{d} = 0$$

Hence, M is a nearly Kahler.

Theorem 3.3

The manifold M is generalized Viessman -gray (GT-manifold) having a flat holomorphic sectional tensor, therefore M has a zero-scalar curvature.

Proof:

According to theorem (3.1) we obtain

$$A^{ad}_{bc} - B^{ah}{}_{c}B_{hb}{}^{d} + \frac{1}{2}\alpha^{[a}_{[b}\delta^{d]}_{c]} + \frac{1}{2(n-1)}\left(-r^{a}_{c}\delta^{d}_{b} - r^{d}_{b}\delta^{a}_{c}\right) + \frac{K}{2(n-1)(2n-1)}\delta^{d}_{b}\delta^{a}_{c} = 0 \quad (3.3)$$

Since M is flat holomorphic sectional curvature tensor then

$$A^{ad}_{bc} = 0 \quad (3.4)$$

$$-B^{ah}{}_{c}B_{hb}{}^{d} + \frac{1}{2}\alpha^{[a}_{[b}\delta^{d]}_{c]} + \frac{1}{2(n-1)}\left(-r^{a}_{c}\delta^{d}_{b} - r^{d}_{b}\delta^{a}_{c}\right) + \frac{K}{2(n-1)(2n-1)}\delta^{d}_{b}\delta^{a}_{c} = 0 \quad (3.5)$$

Antisymmetrizing and symmetries (3.5) by the indexes (a,h), there are

$$\frac{1}{2}\alpha^{[a}_{[b}\delta^{d]}_{c]} + \frac{1}{2(n-1)}\left(-r^{a}_{c}\delta^{d}_{b} - r^{d}_{b}\delta^{a}_{c}\right) + \frac{K}{2(n-1)(2n-1)}\delta^{d}_{b}\delta^{a}_{c} = 0 \quad (3.6)$$

By indexes, equation (3.6) contracts (a, b), and (d, c) we get
$\frac{1}{2}\alpha^{[a}_{[a}\delta^{d]}_{d]} + \frac{1}{2(n-1)}(-r^{a}_{d}\delta^{d}_{a} - r^{d}_{a}\delta^{a}_{d}) + \frac{K}{2(n-1)(2n-1)}\delta^{d}_{a}\delta^{a}_{d} = 0$ (3.7)

Antisymmetrizing and symmetrizing (3.7) by index (a, d) we get K/2(n-1)(2n-1) =O

Hence, M has a zero-scalar curvature.

Theorem 3.4

Let M is generalized Viessman- gray manifold having a flat Lie from, hence M is conharmonic Para_Kohler manifold.
Proof:

By theorem (3.1) we obtain
$A^{ad}_{bc} - B^{ah}{}_{c}B_{hb}{}^{d} + \frac{1}{2}\alpha^{[a}_{[b}\delta^{d]}_{c]} + \frac{1}{2(n-1)}\left(-r^{a}_{c}\delta^{d}_{b} - r^{d}_{b}\delta^{a}_{c}\right) + \frac{K}{2(n-1)(2n-1)}\delta^{d}_{b}\delta^{a}_{c}$
(3.8)

According to theorem (3.2) we obtain

M is Nearly Kahler
$B^{ah}{}_{c}B_{hb}{}^{d} = 0$ (3.9)

Since M is flat Lie form we get
$\frac{1}{2}\alpha^{[a}_{[b}\delta^{d]}_{c]} = 0$ (3.10)

By theorem (3.3) we get

M has a zero-scalar curvature.
$\frac{K}{2(n-1)(2n-1)}\delta^{d}_{b}\delta^{a}_{c} = 0$ (3.11)
$\frac{1}{2(n-1)}\left(-r^{a}_{c}\delta^{d}_{b} - r^{d}_{b}\delta^{a}_{c}\right) = 0$ (3,12)

Contracting (3¸12) by induce (a¸b), &(d¸c) we obtain
$\frac{1}{2(n-1)}(-r^{a}_{d}\delta^{d}_{a} - r^{d}_{a}\delta^{a}_{d}) = 0$ (3.13)

Antisymmetrizing and symmetries (3.13) by the index(a , d) we have
$\frac{1}{2(n-1)}(-r^{a}_{a}\delta^{a}_{a} - r^{a}_{a}\delta^{a}_{a}) = 0$ (3.14)
$W^{*}{}_{\hat{a}bc\hat{a}}$ =0

Hence, according to the definition (3.2)

M is a conharmonic Para_Kohler manifold.

8. Conclusion

Calculate the components of the generalized conformal Viessman - gray manifold (GT-manifold) using the Weyl tensor. It has also been shown that the manifold is Nearly Kähler manifold (NK manifold), and it has also been shown that the manifold has zero scalar curvature. Finally, the relationship between the generalized Viessman-gray manifold (GT manifold) and the Para_Kahler harmonic manifold is found.

REFERENCES

[1] A. Gray, "Sixteen classes of almost Hermitian manifold and their linear invariants," Ann. Math. Pure Appl., vol. 123, no. 3, pp. 35-58, 1980.

[2] A. Shihab and A. K. Ali, "W-Projective Curvature Tensor of the Locally Conformal Kahler Manifold," AIP Conf. Proc., vol. 2834, no. 1, 2023.

[3] M. A. Mohammed and A. A. Shihab, "Some aspects of geometry for conharmonic curvature of locally conformal Kahler manifold," Scholars J. Phys. Math. Stat., vol. 5, pp. 247-254, 2018.

[4] V. F. Kirichenko, "Differential-Geometry Structures on Manifolds," 2nd ed. Odessa: Printing House, 2013, pp. 458.

[5] A. A. Shihab and A. M. Mohammed, "Generalized Conharmonic Curvature Tensor of the Locally Conformal Kahler Manifold," Turk. J. Comput. Math. Educ., vol. 12, no. 11, 2021.

[6] A. Ahmad and A. A. Shihab, "On generalized conharmonic curvature tensor of the Vaisman-Gray manifold," Tikrit J. Pure Sci., vol. 24, no. 7, pp. 117-121, 2019.

[7] S. Kobayashi and K. Nomizu, "Foundations of differential geometry," New York: John Wiley and Sons, 1994, p. 1993.

[8] Q. Ebtihal, A. Ali, and H. Qasim, "The Concircular curvature tensor of Viasman-Gray1 manifold," Int. J. Psychosoc. Rehabilitation, vol. 24, no. 04, pp. 10147-10157, 2020.

[9] L. Ignatochkina, "New aspects of geometry of Viasman-Gray manifold" [PhD thesis], Moscow State Pedagogical University, Moscow, 2001.

[10] H. M. Abood and I. Abed, "Vanishing g-Bochner Tensor of Viasman-Gray Manifold," J. Math. Comput. Sci., vol. 8, no. 6, pp. 470-481, 2015.