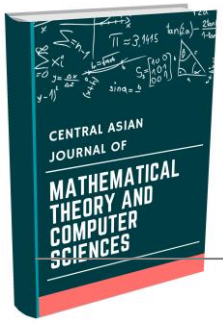

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COMPUTATIONAL MATHEMATICS

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Abstract

Mathematics is an indispensable tool for the efficient treatment of complex real-world problems coming from engineering, computer science, economics, finance, etc. Besides being a science in its own right, mathematics plays a fundamental role in the quantitative areas of practically all other academic disciplines, particularly in the natural sciences, engineering, business, finance, economics, and medicine. Computational mathematics is the blending of computer science with applied mathematics. It may be regarded as a conglomerate of processor programs, which is capable of creating algorithms and methods. As a branch of applied mathematics, computational mathematics is applicable to different fields. This paper is a primer on computational mathematics.

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INTRODUCTION

Mathematics may be regarded as the manipulation of algebra which allows a solution to be found to a given problem. It permeates nearly all facets of life and is a necessary prerequisite for the vast majority of modern technologies. Mathematics and computer science are applicable to different aspects of life. Combining the two of them produces computational mathematics. The major purpose of computational mathematics is to provide easy and direct solutions to difficulties.

Computational mathematics emerged as a branch of applied mathematics by the early 1950s using the computational power of the early digital computers. Computational power allows us to do calculations that you could not perform any other way. Applying computation power to problem solving has led to the development of new ways of conceptualizing, analyzing, and solving problems that are otherwise difficult to tackle. Currently, computational mathematics supports mathematical research in areas where computation plays a central and essential role, emphasizing analysis, development, and implementation of theoretically

justified and efficient algorithms. The goal of computational mathematics is to develop algorithms that solve mathematical problems computationally (i.e. using computers). To be specific, we desire that any algorithm we develop fulfills four primary properties [1]:

- *Accuracy:* An accurate algorithm is able to return a result that is numerically very close to the correct, or analytical, result.
- *Efficiency:* An efficient algorithm is able to quickly solve the mathematical problem with reasonable computational resources.
- *Robustness:* A robust algorithm works for a wide variety of inputs x.
- *Stability:* A stable algorithm is not sensitive to small changes in the input x.

Computational mathematics is concerned with the study of algorithms (or numerical methods) for the solution of computational problems in science, engineering, economics, finance, and other fields. The research areas in computational mathematics cover a broad spectrum, including differential equations, inverse problems, optimization, nonlinear dynamics, computational biology, numerical analysis, fluid mechanics, materials science, computational electromagnetics, acoustics, statistics, signal and image processing, probability theory and stochastic analysis, dynamical systems, and geometric mechanics [2]. Figure 1 shows a group in computational mathematics at the University of Oslo [3].

APPLICATIONS OF COMPUTATIONAL MATHEMATICS

Computational mathematics is a branch of applied mathematics devoted to the interaction between mathematics and computer computation. As a branch of applied mathematics, computational mathematics is applicable to different fields where mathematics is useful. These fields include the following [4]:

- *Engineering:* Computational mathematics is applied to solve technical problems.
- *Numerical Techniques:* These are used in scientific computing and applied in areas such as algebra and equations. The computational mathematics designs and analyzes numerical algorithms and answers fundamental questions about the underlying physics. MATLAB, Mathematica, and Maple software are packages for numerical computation and graphics. Figure 2 displays a result from numerical analysis [5].
- *Computation Simulation:* Computer simulations are used in several important fields. It involves solving mathematical problems by computer simulation as opposed to analytic methods. Computer simulation is regarded as the third pillar of science, complementing theory and experiment. For example, it may involve the virtual construction and modeling of airplanes, satellites, automobiles, and ships. It uses tools like Monte Carlo Methods, Markov Chains, and queuing theory. Monte Carlo Methods are used to introduce randomness. An example of computer simulation using MATLAB is shown in Figure 3 [6].
- *Mathematical Models:* These are our expression of the understanding of the world. They are essential for interpreting the results of scientific experiments, as well as in the design of products, from airplanes to cell phones. Ordinary differential equations are often used for mathematically model problems in many fields. For example, mathematical economics use mathematical models in economics, finance and, accounting.
- *Computational Linguistics*: This refers to the use of mathematical and computer methods in what is known as natural expressions. Linguistics is a discipline that investigates the cognitive and social aspects of human language.
- *Social Sciences:* This adopts a modern scientific approach that combines knowledge from mathematical modeling with various aspects of social science. Special algorithms can be created to simulate an artificial society and a detailed analysis can subsequently be used to project social realities.

- *Financial Mathematics:* This is applied mathematics used to analyze and solve problems related to financial markets. Any informed market participant would exploit an opportunity to make a profit without risk of loss.

DEGREES IN COMPUTATIONAL MATHEMAICS

The computational mathematics degree is usually offered by the department of mathematics. The program emphasizes the use of computers as tools to solve mathematically modeled real-world and data-enabled problems. Some institutions offer online programs. Students in computational mathematics are expected to combine their basic mathematical studies with deep involvement in some field of application. It is the goal of the program to equip students with the necessary skills so that they will become professionals in the specific fields. A student will graduate with a strong foundation in math coupled with the computing skills required by many of today's applications. You will learn how to use software such as R, Python, SAS, and MATLAB to analyze data and draw conclusions. Job prospects vary by field, but you will be prepared for a strong job market. You can be employed at some of the most prestigious tech companies, solving problems using math and computer science.

The following degree programs are offered in computational mathematics.

1. *Bachelor's Degree*: This program develops analytical and problem-solving skills in computing, mathematics, statistics, and science. It provides a solid foundation in the application of mathematics across disciplines that require quantitative analysis. It provides students with a strong mathematical foundation in problem solving and mathematical proof. Graduates pursue careers in a range of industries such as banks, educational institutions, insurance companies, medical research, and stock markets. The computational mathematic program have the following goals [7]:

1. To develop problem-solving skills.
2. To develop inductive and deductive skills in reasoning.
3. To understand the significance of central mathematical theorems and their applications.
4. To appreciate the precision and breadth presented in mathematical theories.
5. To develop and foster abstract mathematical thinking.
6. To be able to reason and compute with mathematical structures, make a conjecture and prove it, generalize, analyze, and abstract a result.
7. To explore the consequences of a general mathematical result in concrete situations.
8. To apply mathematical thinking to real-world situations.
9. To be able to understand, read, interpret, and eventually generate mathematical proofs and examples.
10. To recognize the roles of axiomatic systems and proofs in different branches of mathematics, such as analysis, discrete mathematics, algebra, and geometry.
11. To be able to utilize technology, including computer algebra systems, to solve problems numerically, symbolically, and graphically.
12. To be able to design and apply algorithms to solve problems numerically, algebraically, and graphically.
13. To acquire the skills and confidence to learn new mathematical knowledge as becomes necessary in the course of a lifetime.
14. To build mathematical foundations for success in other disciplines.
15. To understand the principal concepts of the calculus.
16. To build conceptual understanding of sets and functions at various levels.

17. To obtain the mathematical skills needed for the job market (actuarial, financial, or other).
18. In preparation for a career in teaching, to understand the mathematics that will be taught at a profound level and from many points of view.
19. To be able to communicate orally and in writing in the language of mathematics.
20. To gain a familiarity with the history of mathematics.
21. To understand the basic concepts of probability and statistics.
22. To prepare students for graduate study in mathematics.
23. To prepare for careers outside of teaching.

2. *Master Degree:* The Master's program provides graduate-level education in computational mathematics. It prepares students for advanced industrial positions or doctoral studies. Students acquire skills in advanced mathematics and computer simulation that are in demand in several important fields. The curriculum is intended to give the student a working knowledge of several areas of applied mathematics, which may include a specific area of application, in order to prepare for a productive career in industry. Students may choose to complete a thesis or choose a non-thesis option depending on the institution [8]. The program emphasizes applied and theoretical aspects of computational mathematics, so you can [9]:

- Gain high demand skills to advance in industries that rely on mathematical modeling and computational science
- Solve probability problems, including finding properties of distribution functions
- Apply statistical software confidently
- Interpret and solve simple regression models
- Understand the distinction between axioms, definitions, and theorems
- Learn on your terms and select a flexible course schedule in a format that works best for you
- Figure 4 shows some graduate students specializing in computational mathematics at Swansea University [10].

3. *Doctoral Degree:* Some mathematics department offer an interdisciplinary program of graduate study in computational mathematics leading to the PhD degree. The program focuses on applied mathematics, numerical analysis, and scientific computation. This program is designed to give students a thorough training in fundamental computational, develop their research ability in a specific application field, and prepare them for 21st century jobs in academia, national laboratories, or industry. Applications fields include a wide range of areas such as fluid mechanics, materials science, and mathematical biology, engineering applications, image processing, and mathematical finance [1]. The computational mathematics doctoral program has research priorities on the following themes: mathematical modeling of climate-ocean-atmosphere processes, compressible flows, numerical methods for partial differential equations, and the application of machine learning and AI in these disciplines [11]. Doctoral student is often required to conduct original research and write a dissertation based on their work.

BENEFITS AND CHALLENGES

Computational mathematics involves mathematical research in areas of science and engineering where computing plays a central and essential role. It is closely connected with a variety of other mathematical branches. It represents an incredible development. Training in computational mathematics is essential for future careers in research and development in academia, national laboratories, or in industry. Professionals in computational mathematics are needed to assist engineers, scientists, and managers in the formulation of complex problems and in selecting the analytical methods and software appropriate for their solutions. These professionals need to interpret the results of computations for others.

The advanced principles and technologies of computational mathematics should be used to address challenges faced by the global community. The application of such principles and technologies enables the study and modeling of various phenomena of the real world using intelligent software and hardware platforms.

Computational mathematics provides the computational and mathematical models that record and evaluate data and make predictions. Its only limitations become the minds of individuals and at the level of physical development that computers can present, limitations that sooner or later can be solved.

CONCLUSION

If you are seeking a career that draws on your knowledge of mathematics to solve problems in business, science, and engineering, you should consider a degree program on computational mathematics. Some of the programs are offered online. You will explore core topics in computing structures, equations, and algorithms. The programs are designed to prepare future researchers and scientists for 21st century careers, increase students' knowledge and expertise in specific fields, and train them to meet the shortage of skilled human resources in academia, industry, and national laboratories.

A fundamental change is taking place in the role of computational mathematics. Areas such as machine learning, bioinformatics, network science, and medical imaging are increasingly making use of computation and require new computational tractable methods. More information about computational mathematics can be found in the books in [12-39] and the following related journals:

- *Applied and Computational Mathematics*
- *Advances in Computational Mathematics*
- *Advances in Computational Mathematics and Modelling*
- *Computational and Mathematical Methods*
- *Computational Mathematics and Mathematical Physics*
- *Computational Methods in Applied Mathematics*
- *Journal of Computational Mathematics*
- *Journal of Computational Mathematics and Data Science*
- *Journal of Applied and Computational Mathematics*
- *Journal of Advances in Applied & Computational Mathematics*
- *Journal of Mathematical Techniques and Computational Mathematics*
- *International Journal of Applied and Computational Mathematics*
- *International Journal of Pure and Computational Mathematics*

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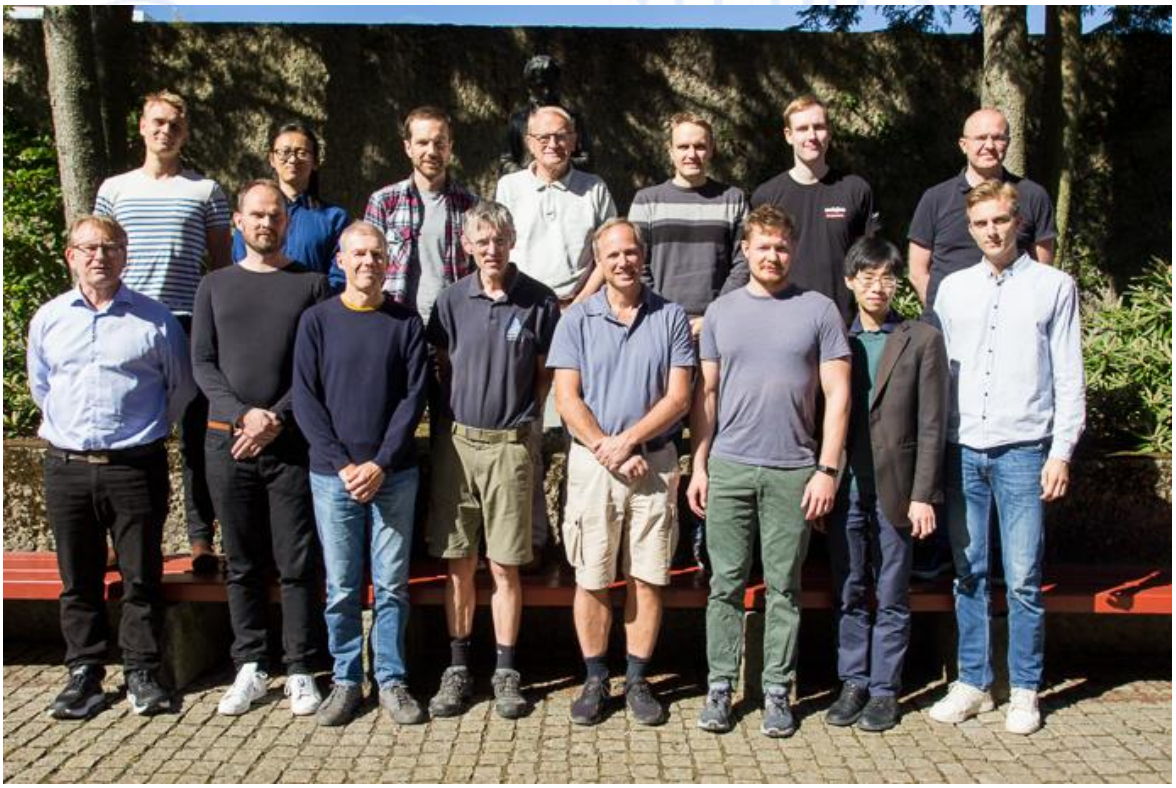

Figure 1 A group in computational mathematics [3].

Figure 2 Result from numerical analysis [5]

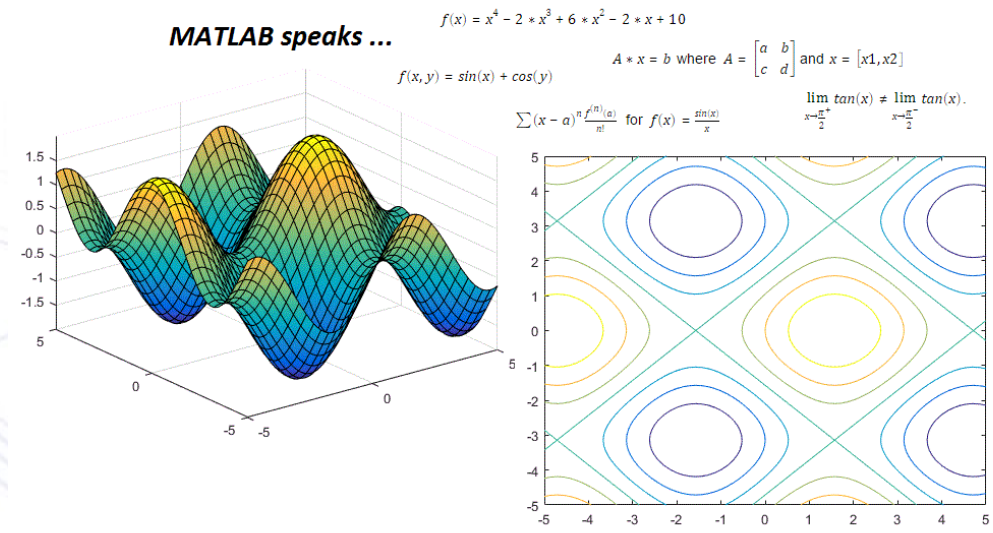

Figure 3 An example of computer simulation using MATLAB [6].

Figure 4 Some graduate students specializing in computational mathematics [10].