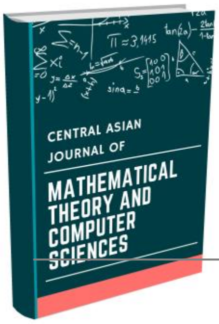

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Application of Kepler's Law in Satellite Motion

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Abstract

In astronomy, Kepler's laws of planetary motion, published by Johannes Kepler between 1609 and 1619, describe the orbits of planets around the Sun. The laws modified the heliocentric theory of Nicolaus Copernicus, replacing its circular orbits and epicycles with elliptical trajectories, and explaining how planetary velocities vary. The three laws state that:[1][2]

1. *The orbit of a planet is an ellipse with the Sun at one of the two foci.*
2. *A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time.*
3. *The square of a planet's orbital period is proportional to the cube of the length of the semi-major axis of its orbit.*

The elliptical orbits of planets were indicated by calculations of the orbit of Mars. From this, Kepler inferred that other bodies in the Solar System, including those farther away from the Sun, also have elliptical orbits. The second law helps to establish that when a planet is closer to the Sun, it travels faster. The third law expresses that the farther a planet is from the Sun, the slower its orbital speed, and vice versa.

Isaac Newton showed in 1687 that relationships like Kepler's would apply in the Solar System as a consequence of his own laws of motion and law of universal gravitation.

A more precise historical approach is found in Astronomia nova and Epitome Astronomiae Copernicanae.

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INTRODUCTION

Johannes Kepler's laws improved the model of Copernicus. According to Copernicus:[3][4]

1. The planetary orbit is a circle with epicycles.
2. The Sun is approximately at the center of the orbit.
3. The speed of the planet in the main orbit is constant.

Despite being correct in saying that the planets revolved around the Sun, Copernicus was incorrect in defining their orbits. Introducing physical explanations for movement in space [1,2,3]beyond just geometry, Kepler correctly defined the orbit of planets as follows:

1. The planetary orbit is not a circle with epicycles, but an ellipse.
2. The Sun is not at the center but at a focal point of the elliptical orbit.
3. Neither the linear speed nor the angular speed of the planet in the orbit is constant, but the area speed (closely linked historically with the concept of angular momentum) is constant.

The eccentricity of the orbit of the Earth makes the time from the March equinox to the September equinox, around 186 days, unequal to the time from the September equinox to the March equinox, around 179 days.

Nomenclature

It took nearly two centuries for the current formulation of Kepler's work to take on its settled form. Voltaire's Eléments de la philosophie de Newton (Elements of Newton's Philosophy) of 1738 was the first publication to use the terminology of "laws".[6][7] The Biographical Encyclopedia of Astronomers in its article on Kepler (p. 620) states that the terminology of scientific laws for these discoveries was current at least from the time of Joseph de Lalande.[8] It was the exposition of Robert Small, in An account of the astronomical discoveries of Kepler (1814) that made up the set of three laws, by adding in the third.[9] Small also claimed, against the history, that these were empirical laws, based on inductive reasoning.[7][10]

Further, the current usage of "Kepler's Second Law" is something of a misnomer. Kepler had two versions, related in a qualitative sense: the "distance law" and the "area law". The "area law" is what became the Second Law in the set of three; but Kepler[4,5,6] did himself not privilege it in that way.[11]

Kepler published his first two laws about planetary motion in 1609,[12] having found them by analyzing the astronomical observations of Tycho Brahe.[13] Kepler's third law was published in 1619.[16][14] Kepler had believed in the Copernican model of the Solar System, which called for circular orbits, but he could not reconcile Brahe's highly precise observations with a circular fit to Mars' orbit – Mars coincidentally having the highest eccentricity of all planets except Mercury.[17] His first law reflected this discovery.

In 1621, Kepler noted that his third law applies to the four brightest moons of Jupiter.[Nb 1] Godefroy Wendelin also made this observation in 1643.[Nb 2] The second law, in the "area law" form, was contested by Nicolaus Mercator in a book from 1664, but by 1670 his Philosophical Transactions were in its favour.[18][19] As the century proceeded it became more widely accepted.[20] The reception in Germany changed noticeably between 1688, the year in which Newton's Principia was published and was taken to be basically Copernican, and 1690, by which time work of Gottfried Leibniz on Kepler had been published[7,8,9]

Newton was credited with understanding that the second law is not special to the inverse square law of gravitation, being a consequence just of the radial nature of that law, whereas the other laws do depend on the inverse square form of the attraction. Carl Runge and Wilhelm Lenz much later identified a symmetry principle in the phase space of planetary motion (the orthogonal group O(4) acting) which accounts for the first and third laws in the case of Newtonian gravitation, as conservation of angular momentum does via rotational symmetry for the second law.[22]

Formulary

The mathematical model of the kinematics of a planet subject to the laws allows a large range of further calculations.

First law

The orbit of every planet is an ellipse with the Sun at one of the two foci.

Second law

A line joining a planet and the Sun sweeps out equal areas during equal intervals of time.[23]

DISCUSSION

Third law

The ratio of the square of an object's orbital period with the cube of the semi-major axis of its orbit is the same for all objects orbiting the same primary.

This captures the relationship between the distance of planets from the Sun, and their orbital periods.

Kepler enunciated in 1619[16] this third law in a laborious attempt to determine what he viewed as the "music of the spheres" according to precise laws, and express it in terms of musical notation.[24] It was therefore known as the harmonic law.[25]

The following table shows the data used by Kepler to empirically derive his law:

Data used by Kepler (1618)			
Planet	Mean distance to sun (AU)	Period (days)	(10^{-6} AU^3/day^2)
Mercury	0.389	87.77	7.64
Venus	0.724	224.70	7.52
Earth	1	365.25	7.50
Mars	1.524	686.95	7.50
Jupiter	5.20	4332.62	7.49
Saturn	9.510	10759.2	7.43

Upon finding this pattern Kepler wrote:[26]

I first believed I was dreaming... But it is absolutely certain and exact that the ratio which exists between the period times of any two planets is precisely the ratio of the 3/2th power of the mean distance.

—translated from Harmonies of the World by Kepler (1619)

Log-log plot of period T vs semi-major axis a (average of aphelion and perihelion) of some Solar System orbits (crosses denoting Kepler's values) showing that a^3/T^2 is constant (green line)

For comparison, here are modern estimates:

Modern data (Wolfram Alpha Knowledgebase 2018)			
Planet	Semi-major axis (AU)	Period (days)	(10^{-6} AU^3/day^2)
Mercury	0.38710	87.9693	7.496
Venus	0.72333	224.7008	7.496
Earth	1	365.2564	7.496
Mars	1.52366	686.9796	7.495
Jupiter	5.20336	4332.8201	7.504
Saturn	9.53707	10775.599	7.498
Uranus	19.1913	30687.153	7.506
Neptune	30.0690	60190.03	7.504

Planetary acceleration

Isaac Newton computed in his Philosophiæ Naturalis Principia Mathematica the acceleration of a planet moving according to Kepler's first and second laws.

1. The direction of the acceleration is towards the Sun.

2. The magnitude of the acceleration is inversely proportional to the square of the planet's distance from the Sun (the inverse square law).

This implies that the Sun may be the physical cause of the acceleration of planets. However, Newton states in his Principia that he considers forces from a mathematical point of view, not a physical, thereby taking an instrumentalist view.[27] Moreover, he does not assign a cause to gravity.[28]

Newton defined the force acting on a planet to be the product of its mass and the acceleration (see Newton's laws of motion). So:

1. Every planet is attracted towards the Sun.
2. The force acting on a planet is directly proportional to the mass of the planet and is inversely proportional to the square of its distance from the Sun.[10,11]

The Sun plays an unsymmetrical part, which is unjustified. So he assumed, in Newton's law of universal gravitation:

1. All bodies in the Solar System attract one another.
2. The force between two bodies is in direct proportion to the product of their masses and in inverse proportion to the square of the distance between them.

As the planets have small masses compared to that of the Sun, the orbits conform approximately to Kepler's laws. Newton's model improves upon Kepler's model, and fits actual observations more accurately.

RESULTS

Kepler used his two first laws to compute the position of a planet as a function of time. His method involves the solution of a transcendental equation called Kepler's equation.

The procedure for calculating the heliocentric polar coordinates (r,θ) of a planet as a function of the time t since perihelion, is the following five steps:

1. Compute the mean motion n = (2π rad)/P, where P is the period.
2. Compute the mean anomaly M = nt, where t is the time since perihelion.

In celestial mechanics, a Kepler orbit (or Keplerian orbit, named after the German astronomer Johannes Kepler) is the motion of one body relative to another, as an ellipse, parabola, or hyperbola, which forms a two-dimensional orbital plane in three-dimensional space. A Kepler orbit can also form a straight line. It considers only the point-like gravitational attraction of two bodies, neglecting perturbations due to gravitational interactions with other objects, atmospheric drag, solar radiation pressure, a non-spherical central body, and so on. It is thus said to be a solution of a special case of the two-body problem, known as the Kepler problem. As a theory in classical mechanics, it also does not take into account the effects of general relativity. Keplerian orbits can be parametrized into six orbital elements in various ways.

In most applications, there is a large central body, the center of mass of which is assumed to be the center of mass of the entire system. By decomposition, the orbits of two objects of similar mass can be described as Kepler orbits around their common center of mass, their barycenter.

From ancient times until the 16th and 17th centuries, the motions of the planets were believed to follow perfectly circular geocentric paths as taught by the ancient Greek philosophers Aristotle and Ptolemy. Variations in the motions of the planets were explained by smaller circular paths overlaid on the larger path (see epicycle). As measurements of the planets became increasingly accurate, revisions to the theory were proposed. In 1543, Nicolaus Copernicus published a heliocentric model of the Solar System, although he still believed that the planets traveled in perfectly circular paths centered on the Sun.[1]

Development of the laws

In 1601, Johannes Kepler acquired the extensive, meticulous observations of the planets made by Tycho Brahe. Kepler would spend the next five years trying to fit the observations of the planet Mars to various curves. In 1609, Kepler published the first two of his three laws of planetary motion. The first law states:

The orbit of every planet is an ellipse with the sun at a focus.

More generally, the path of an object undergoing Keplerian motion may also follow a parabola or a hyperbola, which, along with ellipses, belong to a group of curves known as conic sections. Mathematically, the distance between a central body and an orbiting body can be expressed as:

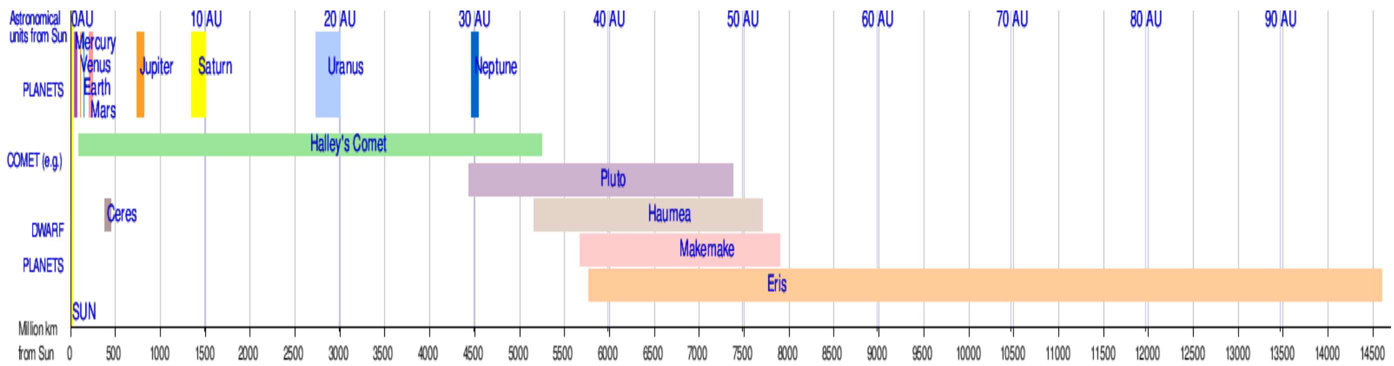
From the laws of motion and the law of universal gravitation, Newton was able to derive Kepler's laws, which are specific to orbital motion in astronomy. Since Kepler's laws were well-supported by observation data, this consistency provided strong support of the validity of Newton's generalized theory, and unified celestial and ordinary mechanics. These laws of motion formed the basis of modern celestial mechanics until Albert Einstein introduced the concepts of special and general relativity in the early 20th century. For most applications, Keplerian motion approximates the motions of planets and satellites to relatively high degrees of accuracy and is used extensively in astronomy and astrodynamics.

Any Keplerian trajectory can be defined by six parameters. The motion of an object moving in three-dimensional space is characterized by a position vector and a velocity vector. Each vector has three components, so the total number of values needed to define a trajectory through space is six. An orbit is generally defined by six elements (known as Keplerian elements) that can be computed from position and velocity, three of which have already been discussed. These elements are convenient in that of the six, five are unchanging for an unperturbed orbit (a stark contrast to two constantly changing vectors). The future location of an object within its orbit can be predicted and its new position and velocity can be easily obtained from the orbital elements.[12,13]

Two define the size and shape of the trajectory:

- Semimajor axis
- Eccentricity

In the Solar System, planets, asteroids, most comets, and some pieces of space debris have approximately elliptical orbits around the Sun. Strictly speaking, both bodies revolve around the same focus of the ellipse, the one closer to the more massive body, but when one body is significantly more massive, such as the sun in relation to the earth, the focus may be contained within the larger massing body, and thus the smaller is said to revolve around it. The following chart of the perihelion and aphelion of the planets, dwarf planets, and Halley's Comet demonstrates the variation of the eccentricity of their elliptical orbits. For similar distances from the sun, wider bars denote greater eccentricity. Note the almost-zero eccentricity of Earth and Venus compared to the enormous eccentricity of Halley's Comet and Eris.



Distances of selected bodies of the Solar System from the Sun. The left and right edges of each bar correspond to the perihelion and aphelion of the body, respectively, hence long bars denote high orbital eccentricity. The radius of the Sun is 0.7 million km, and the radius of Jupiter (the largest planet) is 0.07 million km, both too small to resolve on this image.

CONCLUSION

In astrodynamics or celestial mechanics, an elliptic orbit or elliptical orbit is a Kepler orbit with an eccentricity of less than 1; this includes the special case of a circular orbit, with eccentricity equal to 0. In a stricter sense, it is a Kepler orbit with the eccentricity greater than 0 and less than 1 (thus excluding the circular orbit). In a wider sense, it is a Kepler orbit with negative energy. This includes the radial elliptic orbit, with eccentricity equal to 1.

In a gravitational two-body problem with negative energy, both bodies follow similar elliptic orbits with the same orbital period around their common barycenter. Also the relative position of one body with respect to the other follows an elliptic orbit.

Examples of elliptic orbits include Hohmann transfer orbits, Molniya orbits, and tundra orbits.

- For a given semi-major axis the specific orbital energy is independent of the eccentricity.

Using the virial theorem we find:

- the time-average of the specific potential energy is equal to -2ε
  - ✓ the time-average of r^{-1} is a^{-1}
- the time-average of the specific kinetic energy is equal to ε

The state of an orbiting body at any given time is defined by the orbiting body's position and velocity with respect to the central body, which can be represented by the three-dimensional Cartesian coordinates (position of the orbiting body represented by x, y, and z) and the similar Cartesian components of the orbiting body's velocity. This set of six variables, together with time, are called the orbital state vectors. Given the masses of the two bodies they determine the full orbit. The two most general cases with these 6 degrees of freedom are the elliptic and the hyperbolic orbit. Special cases with fewer degrees of freedom are the circular and parabolic orbit.

Because at least six variables are absolutely required to completely represent an elliptic orbit with this set of parameters, then six variables are required to represent an orbit with any set of parameters. Another set of six parameters that are commonly used are the orbital elements.

The Babylonians were the first to realize that the Sun's motion along the ecliptic was not uniform, though they were unaware of why this was; it is today known that this is due to the Earth moving in an elliptic orbit around the Sun, with the Earth moving faster when it is nearer to the Sun at perihelion and moving slower when it is farther away at aphelion.[8]

In the 17th century, Johannes Kepler discovered that the orbits along which the planets travel around the Sun are ellipses with the Sun at one focus, and described this in his first law of planetary motion. Later, Isaac Newton explained this as a corollary of his law of universal gravitation.[14]

REFERENCES

1. Copernicus. pp 513–514
2. Bate, Mueller, White. pp 177–181
3. "NASA website". Archived from the original on 16 February 2011. Retrieved 12 August 2012.
4. El'Yasberg "Theory of flight of artificial earth satellites", Israel program for Scientific Translations (1967)
5. Bate, Roger; Mueller, Donald; White, Jerry (1971). Fundamentals of Astrodynamics. Dover Publications, Inc., New York. ISBN 0-486-60061-0.

6. Copernicus, Nicolaus (1952), "Book I, Chapter 4, The Movement of the Celestial Bodies Is Regular, Circular, and Everlasting-Or Else Compounded of Circular Movements", On the Revolutions of the Heavenly Spheres, Great Books of the Western World, vol. 16, translated by Charles Glenn Wallis, Chicago: William Benton, pp. 497–838
7. Bate, Mueller, White (1971). Fundamentals of Astrodynamics (First ed.). New York: Dover. pp. 11–12. ISBN 0-486-60061-0.
8. Lissauer, Jack J.; de Pater, Imke (2019). Fundamental Planetary Sciences: physics, chemistry, and habitability. New York, NY, USA: Cambridge University Press. pp. 29–31. ISBN 9781108411981.
9. Bate, Mueller, White (1971). Fundamentals of Astrodynamics (First ed.). New York: Dover. p. 33. ISBN 0-486-60061-0.
10. Bate, Mueller, White (1971). Fundamentals of Astrodynamics (First ed.). New York: Dover. pp. 27–28. ISBN 0-486-60061-0.
11. Bate, Mueller, White (1971). Fundamentals of Astrodynamics (First ed.). New York: Dover. p. 15. ISBN 0-486-60061-0.
12. Bate, Mueller, White (1971). Fundamentals of Astrodynamics (First ed.). New York: Dover. p. 18. ISBN 0-486-60061-0.
13. Bate, Mueller, White (1971). Fundamentals of Astrodynamics (First ed.). New York: Dover. p. 17. ISBN 0-486-60061-0.
14. David Leverington (2003), Babylon to Voyager and beyond: a history of planetary astronomy, Cambridge University Press, pp. 6–7, ISBN 0-521-80840-5