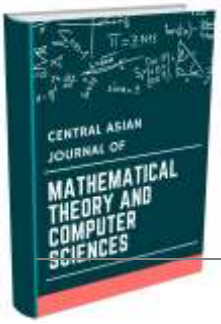

CENTRAL ASIAN JOURNAL OF MATHEMATICAL THEORY AND COMPUTER SCIENCES
https://cajmtcs.centralasianstudies.org

Volume: 04 Issue: 03 | March 2023 **ISSN: 2660-5309**

The Nominalist Theories of Number: A Peano Arithmetic Critique

Etorobong Godwin Akpan

Department of Philosophy, University of Port Harcourt
Email: etorobong.akpan@gmail.com

Abstract

The paper is concerned with the study of the nominalist theories of number. The objective of the study is to evaluate nominalist theories against the backdrop of their capacity to adequately interpret Peano Arithmetic, having accepted the arithmetic as a standard and a fruitful formalism for general arithmetic. Two strands of nominalism are considered, namely; formalism and fictionalism. Using the method of content analysis of texts and publications on the subject matter, it has been shown that formalism denies symbolic transcendence and the necessity of first order logic in a system model, while fictionalism claims that mathematical statements are vacuous even in the face of the use of bound variables. The claims of the two strands of nominalism, as well as the general claim of nominalism itself, are contrary to the nature of Peano Arithmetic in particular and arithmetic in general. Hence, nominalism cannot provide an adequate theory of number that can satisfy the interpretation of general arithmetic.

ARTICLE INFO

Article history:
Received 28 Jan 2023
Revised form 3 Mar 2023
Accepted 1 Apr 2023

Key words:
Nominalism, Peano Arithmetic, Fictionalism, Formalism, Model, Number.

---***---

INTRODUCTION

It was an indubitable fact among mathematicians in the foundations of mathematics that the only sure approach to overcoming the paradoxes in *Analysis*, was to control the meaning of mathematical concepts. Reductionism was understood to be the proven means by which this feat could be achieved. Reductionism involves the reduction of the whole mathematical corpus to a set of terms, a few postulates stated on the basis of the terms, and rules that would determine the relationship between both the terms and the postulates. This approach is christened "formalism," and it is at the heart of all the programmes in the foundations of mathematics. In an attempt to address the paradoxes by using the above approach, the foundations of mathematics became a cemetery of failed and unsuccessful formalisms.

It took the genius of Giuseppe Peano (1967) to provide a formal system that would break the jinx of failed formalisms and then adequately carry the weight of reductionism for all mathematics while equally overcoming the paradoxes associated with *Analysis*. The system developed by Giuseppe Peano is known in modern-day mathematics as *Peano Arithmetic,* and it would subsequently be referred to in the essay as *PA*. Aside from categorically modelling arithmetic, some key features of the system are its simplicity in the sense that it is formulated in first-order predicate logic, its accuracy, and its fruitfulness in bringing forth consistent mathematical results. By accepting the validity of first-order predicate logic, the system is free from the formalist problem of incompleteness, as shown by Kurt Gödel (1986).

Bertrand Russell (1992) has argued that *PA* is an accurate representation of the mathematical body of knowledge. Consequently, any formalization of the mathematical theory that is not isomorphic to *PA* is not adequate as a foundation of mathematics. The basic concepts of *PA* are zero, number, and successor. It could be argued that any interpretation of these concepts in mathematical theory must be such that the interpreting system permits its concepts to satisfy the elliptical spaces of zero, number, and successor in *PA*; otherwise, the interpretation is inaccurate and inappropriate. The implication of the foregoing argument is that the satisfaction of *PA* is the satisfaction of the interpretation of the mathematical theory. Hence, the interpretation of *PA* could be used as a yardstick for measuring the adequacy of any interpretation of the general mathematical theory by schools in the foundations of mathematics. Nominalism is one such school.

It is the purpose of the paper to examine nominalist theories of number against the backdrop of whether or not their interpretation of the concept is consistent with its behaviour in the formalism or isomorphic models of *PA*. To achieve this objective, the paper adopts the method of content analysis. The argument of the study would be guided by the following thesis statements: It is the thesis of the paper that any adequate philosophy of mathematics that seeks to satisfy the semantic requirements of both *PA* and general arithmetic must do so taking into consideration that numbers are essentially ordinal and only cardinal in application; otherwise, such a philosophical interpretation is inadequate. Besides, the use of bound variables in *PA* equally suggests that its formal system is not vacuous. Hence, any vacuous interpretation of *PA* is an inaccurate interpretation of the system.

THE CONCEPT OF NOMINALISM

Nominalism is from the Latin roots "*nominalis*" and "nomen," which means name (Finedictionary Online). Nominalism as a concept first appeared as a doctrine during the medieval discussion on the ontological status of universal concepts. It was an attempt to champion an anti-realist argument against those positing esoteric entities in a non-material realm. In the debate concerning the ontological status of universal concepts, nominalism is the argument that universals are just names. Consequently, there is nothing in the mind, in an esoteric realm, or in the physical universe that corresponds to general terms. A vehement anti-realist position with a corresponding sponsorship of nominalism is identified with William of Ockham.

William of Ockham had argued that realism, which posits the existence of extra-mental entities corresponding to general terms, is the "worst error in philosophy" (Adam, 1977).

He refused to commit himself to the ontology of general terms as he insisted: "that only particular substances and qualities, not quantities and relations, are distinct real things" (Adam, 1977, p.1). Ockham dismissed universal concepts as mere names used to refer to particulars that stand in a definite relation to one another. Hence, universals have no extra-mental or intra-mental existence. The definition of nominalism in the paper is not oblivious of the recent distorted use of the term in Harvard, where nominalism is not the denial of general terms but abstract objects (Legg, 2011).

NOMINALIST INTERPRETATION OF MATHEMATICAL THEORY

Willard van Orman Quine (1971) has pointed out that the nominalist thesis has been revived by formalism in the foundations of mathematics in the hands of David Hilbert (Quine, *From a Logical* 14). Formalism is the argument that mathematics is a science of meaningless concepts. But it is important to note that, apart from formalism, there is another version of nominalism in the foundations of mathematics called fictionalism. Nominalism, in all its versions in the philosophy of mathematics, is a refutation of realism by a somewhat unwillingness to make ontological commitments to some mathematical entities. Nominalism involves the denial of the actual existence of mathematical entities such as "numbers, points, functions, sets, and so on" (Shapiro, 2016, p. 21). For the nominalist, there is a sense in which we can make meaning of mathematics without necessarily "assuming a mathematical ontology" (Shapiro, 2016, p. 21).

FORMALISM AS NOMINALISM

According to formalism, a subset of nominalism, mathematics is an activity focused on the development of chess-like games. The characters in the play are the symbols on paper, which respond to set rules. There is no going beyond this game to find meaning for mathematical concepts. From the point of view of formalists, mathematics is a game of meaningless symbols. Formalists acknowledge the validity of logic as formal.

Formalists take numbers at face value as meaningless symbols. There is no going beyond signs to ask for the meaning of numbers. For as David Hilbert, one of the formalist philosophers of mathematics, states, "In the beginning there was a sign" (Resnik, 1980, p. 82). The signs are the basics for the logical operations that give rise to the mathematical theory. Consequently, the formalists reject Leibniz’s perception of mathematical theory as a pretentious extension of logic. Following Kantian epistemology of mathematics and logic in the *Critique of Pure Reason* (1965), David Hilbert argues that:

... something which is presupposed in the making of logical inferences and in carrying out logical operations is already given in representation...i.e., certain extra-logical concrete objects, which are inductively present as immediate experience, and underlie all thought. If logical thinking is to be secure, these objects must be capable of being exhaustively surveyed in their parts; and the exhibition, the distinction, the succession of their parts, and their arrangement beside each other must be given, with the objects themselves as something that cannot be reduced to anything else or indeed be in any need of such reduction (Korner, 1971, p. 73).

The said representations are, for the formalists, the numerical symbols. Formalism is characterized by an acceptance of the ontology of signs as the foundations of mathematics. Hamilton defines the system by making reference to the root of the concept of formal. According to him:

The word “formal”... is used when referring to a situation where symbols are being used and where the behavior and properties of the symbols are determined completely by a given set of rules. In a formal system, the symbols have no meanings, and in dealing with them we must be careful to assume nothing about their properties other than what is specified in the system (1978, p. 27).

The tradition of absolute identification of numbers with symbols is traceable to Bombelli. It was Bombelli that first used symbols to represent complex numbers. For instance, the square root of two *2* was represented by *‘2’*. Mathematicians were reluctant to accept the invention until Gauss applied it in an elegant mathematical analysis. But Bombelli’s absolute symbolism for numbers was limited to complex numbers.

The first wholesale declaration that numbers were signs, arbitrarily designed by human fiat, appeared in the works of formalists. The new approach was due to advancements in set theory through the method of transfinite induction. Cantor established the existence of the infinite. Since no empirical fact corresponded with the idea of the infinite despite its consistency with finite mathematics, the formalists thought that all mathematical entities were, like them, human creations manipulated by rules. It is this central belief that unites and defines formalism as a school in the philosophy of mathematics. Apart from the central thesis of the school, formalism has many brands.

Historically, formalism divides into three major brands namely:

i. Game formalism, which takes mathematics to be a meaningless, chess-like game in which the symbolism functions as the ‘board and pieces’,

ii. Theory formalism, which treats mathematics as the theory of formal system of symbols,

iii. Finitism, which views part of mathematics as a meaningful theory of certain symbolic object and the remainder as an instrumentalistic extension of the former (Resnik, 1980, p. 54).

Hein and Thomae championed game formalism. Faced with the problem of actual infinity, Hein argued that the symbol and the number are one and the same (Resnik, 1980). Michael Resnik quotes him as follows:

Suppose that I am not satisfied to have nothing but positive rational numbers. I do not answer to the question ‘what is a number?’ by defining number conceptually; say by introducing irrationals as limits, whose existence is presupposed. I defined from the standpoint of a pure formalist and call certain tangible signs numbers. Thus, the existence of these numbers is not in question (Resnik, 1980, p. 55).

It was Thomae, who gave a more systematic structure to game formalism. Unlike Hein, who left the explanation of the mathematical theory to arbitrariness, Thomae argued that mathematics is like a chess game with rules. The numbers are like the board and the pieces, played according to laid down rules. Conventionalism was the undertone of game formalism. Thomae sought to eliminate conventionalism by his demand that the rules be designed to capture the perceptual manifold. He believed that all of mathematics could be derived from just ten signs namely *0, 1, 2, 3,4,5,6,7,8,9* (Resnik, 1980).

Frege and other scholars have however observed that Thomae used more signs than the ten he listed without any indication of how the additional signs were composed. He is also alleged to have used the commutative law without explanation. Furthermore, where will Thomae have got the idea of the perceptual manifold if he did not presuppose empirical foundations for mathematics?

Game formalism deprives symbols of their transcendent essence. Every symbol system is a representation of something beyond itself. For instance, the king in the Chess game represents the ultimate piece on the board. So, the reduction of mathematics to ordinary symbol system without any ontological transcendence, real or imagined makes formalism even more inadequate than any other system for an arithmetic model theoretic programme.

Another brand of formalism is Hilbert's finitism. After David Hilbert's success in the axiomatization of Euclidean geometry, he set out to prove the consistency of all of mathematics. To legitimize the ontology of the infinite, he argued that mathematics has finite and infinite objects (Raatikainen, *Gödel's Incompleteness Theorem* 2). These objects arise from the unary numeral and its production rules (Korner, 1971). The numeral in question is the figure *1* (Korner, 1971). The ontological status of this number for Hilbert is limited to being a token.

Reacting to Frege and Dedekind's logicist programme, Hilbert argued that mathematics cannot be reduced to logic because as Kant has demonstrated, the objects of mathematics are given independently of logic (Hilbert, cited in Benacerraf and Putnam, 2016). Hence, Hilbert recognized the objects of mathematics as the objects of our intuition of space and time. For Hilbert, mathematics cannot reduce to logic, because before logical operations commence some object must be given (Hilbert, cited in Benacerraf and Putnam, 2016). Hilbert contends that these extra-logically given contents are intuited as directly experienced (objects) prior to all thinking (Hilbert, cited in Benacerraf and Putnam, 2016). He believes that the intuited object must be understood in relation to its sequences, differences, properties and contiguities or relations (Hilbert, cited in Benacerraf and Putnam, 2016).

Consequently, the concrete symbols, which result from this intuition is the subject matter of mathematics. These concrete signs are *1, 11, 111* etc. Hilbert argues that this intuited process goes to show that arithmetic is made up of one's (*1's*). Hilbert's commitment to situating this *1's* in conception (Hilbert, cited in Benacerraf and Putnam, 2016) appears not to be a commitment to formalism but to conceptualism or to constructivism. He makes transition from this unique symbol of ones to the traditional numerals. For which reason he argues that there is no going beyond the numerals to asking questions concerning the subject matter of mathematics. The subject matter of mathematics is: "… the concrete symbols themselves whose structure is immediately clear and recognizable" (Hilbert, cited in Benacerraf and Putnam, 2016). The "recognizable structure" referred to above is explained by Hilbert to mean that: "each numerical symbol is intuitively recognizable by the fact that it contains only *1's*. David Hilbert did not take a constructivist approach to explain how the other numerals are possible from our intuition of one in Kantian epistemology. He assumes that Kant has already shown how they are given and therefore mathematics should proceed by not seeking to go beyond them to asking questions about their possibility. The unfortunate situation is that Kant did not show how the numerals are to be constructed from one. Following Kant is assuming that the numerals are concrete and indissolubly givens. It is in this very sense that Hilbert is a formalist because of his literary interpretation of Kant, because by so doing, there is no more to the numerals and their symbols than their concrete appearances as *1's*. This position is collaborated by the following citation from Hilbert: "these numerical symbols which are themselves our subject matter have no significance in themselves" (Hilbert, cited in Benacerraf and Putnam, 2016).

According to him, mathematics is divided into four major parts, namely: "construction, theory, formalism and metamathematics" (Korner, 1971, p. 77). David Hilbert explains that construction is dependent on theory, which expresses the statement of the rules. Thus, mathematics is the actualization of formalism.

Hilbert therefore understands formalism as a system of rules for constructing metamathematics or as an apparatus:

The consideration of the concrete theory alone creates a picture in which the science of mathematics is reducible to number equations. But the science of mathematics... is in no way exhausted by number equation and is not entirely reducible to such. Yet one can assert that it is an apparatus, which in its application to whole number must always yield correct numerical equations (Korner, 1971, p. 76).

The resultant system is a formula game. It is carried out according to certain rules. Hilbert believed that the resultant formal system could be extended through the method of transfinite induction to solve problems concerning the infinite. Raatikainen argues that Hilbert divided his whole project into two major parts. First, all intuitionistic mathematics was to be formalized; next, one should, using only finitistic mathematics, prove the consistency of this comprehensive system; moreover one should show that infinitistic mathematics would never prove infinitistic real sentences that were unprovable by finitistic mathematics (Raatikainen, *Gödel's Incompleteness Theorem*, 2007). Hilbert believed that this would guarantee the safety and reliability of using infinitary methods in mathematics which, after set-theoretic paradoxes, had been discovered was brought to question (Raatikainen, *Gödel's Incompleteness Theorem*, 2007).

It was Haskell Curry in his theory formalism that stretched formalism to its logical conclusion. According to him, mathematics is a theory of formal systems. This understanding of the theory had no plan of proving the consistency of the mathematical theory. So, Curry did not border about the logical consistency of the system, so long as it was formally consistent. He believed that the most important aspect of the formal system is the formalism. The propositions of mathematics are actually meta-propositions in Curry's formalism. They are hypothetical propositions of possible formal systems. Concerning the problem of truth in mathematics, Curry confesses that a proposition is true if it could be shown to be a theorem of a particular formal system.

FICTIONALISM AS NOMINALISM

A version of nominalism in recent times is the one due to Hartry Field. Field's version of nominalism is called fictionalism. For fictionalism, mathematics is likened to the works of fiction, where numbers, points, sets etc., have the same ontological status as Okonkwo and Ikemefuna in Chinua Achebe's novel *Things Fall Apart* (1980). Consequently, the only important elements in mathematical theory are the logical consequences of the theory (Shapiro, 2016). Kicking against the realist thesis, Field argued that mathematical truths are not a priori because there is no content to necessitate the judgment of truth or falsehood. For him, the only serious argument in favour of realism is that offered by Quine and Putnam (Shapiro, 2016).

Fictionalist nominalism is taken far enough as to purpose the elimination of mathematics from scientific theory. It is Field's conviction that any scientific theory expressed with the aid of mathematical language could as well be expressed as a pure physical theory, without the aid of mathematics. Here then is Field's dispensability thesis for mathematics.

Field's fictionalism is not the only orientation in the fictionalist tradition. Burgess has suggested an alternative to the dispensability thesis of Field. Burgess goes utilitarian in his fictionalism, thereby arguing that mathematics is retained on utilitarian grounds. He does not however, make any ontological commitment to mathematical entities (Burgess, *Philosophia Mathematica*, 2016). During the same period S. Hoffman had argued that mathematics is a set of stories with unique characters, who living in a world like ours express their unique tendencies (Hoffman, 2004). Hoffman's fictionalism appears as a realist theory at face value but Hoffman understands the characters in the mathematical narrative to be fictional characters. Despite these later attempts to redefine fictionalism, it is Field's version of the theory that is still the most influential fictionalsim.

PEANO ARITHMETIC (PA) CRITIQUE OF THE NOMINALIST THEORIES OF NUMBER

The present section of the essay would proceed first by examining the basic notions and axioms of *PA* and thereafter engage in the critique of the nominalist theories of number. It is important to note that for ease of understanding and clarity, the version of *PA* that shall be reviewed in the paper is that given by Bertrand Russell in his *Introduction to Mathematical Philosophy* (1992). Russell's presentation of *PA* is as follows:

Professor Peano believed that he could deduce the whole of arithmetic from three primitive notions and five primitive propositions (Russell, *Mathematical Philosophy*, 1998). The notions include: 0, number, successor (Russell, *Mathematical Philosophy*, 1998). From the above three notions, Peano formed the following five propositions:

(1) *0* is a number
(2) The successor of any number is a number
(3) No two numbers have the same successor
(4) *0* is not the successor of any number
(5) And property which belongs to *0*, and also to the successor of every number, which has the property, belongs to all numbers (Russell, *Mathematical Philosophy*, 1998).

The last axiom is called the principle of mathematical induction. Bertrand Russell argues that Peano's arithmetic satisfies arithmetic equations, especially those of ordinal arithmetic. But it lacked a precise meaning of the concept of number, which meaning is what philosophies of mathematics like nominalism seek to fill. As earlier noted, Peano's mathematical system is a reductionist model of the totality of pure mathematics. So, any model theory of Peano's mathematics is a model theory for pure mathematics. Just as any model theory of the latter is also a mega model for the former. Thus, the analysis so far represents a model theory of both general pure mathematical theory and Peano's mathematical system.

` Nominalism, however would fail to provide an adequate model the arithmetic, because of three very important assumptions in its theories, namely;

1. The claim that mathematical symbols are non-transcendent.
2. The view that the laws of mathematics are not necessarily logical principles but mere rules of a formula game.
3. The claim that mathematical statement are non-realist but vacuous statements.

All three assumptions run contrary to the structure and content of general arithmetic and *PA*. For instance, formalism, in general, cannot be used for providing linguistically transcendent model for the mathematical theory, because it advocates the translation of a system from one meaningless symbol system to another. A model that must overcome the limitation of Gödel's incompleteness theorems must be meaningful. Formalism is not such a model. Besides, formalism cannot suffice as an interpretation for Peano Arithmetic because its formalism transcends that of the system. Peano's mathematics is a first-level formalism compared to Hilbert's formalism, which is a higher level formalism, tending towards a pure game. Peano's system accepted the legitimacy of intuitive predicate logic, whereas Hilbert permits no intuitions except in the algorithms.

Analysts have argued that the proof of the ontology of the infinite is not possible in any formula game. Kurt Gödel has also demonstrated that the consistency proof for the formula system is not possible, because of what he calls axiomatic incompleteness. The incompleteness theorem of Kurt Gödel showed that no formal system can prove its own consistency. The logical consistency of every formal system is inputted from outside the system by intuition (Raatikainen, *Synthese*, 2007). It can therefore not be formalizable in the system. One way of consistency proof is by modeling. But the formalists refused all modeling for their system, despite their quest for both formal and logical consistencies.

Fictionalism or formalism or any nominalist theory will fail to model Peano arithmetic as an interpretation because the translation of Peano arithmetic into a logical theory with the aid of first order logic shows the assumption of the validity and application of bound variables in the system. Bound variables are variables, which were initially free but which have been made to range over a given domain. Hence, it would amount to a contradiction to assume that a system with bound variables is vacuous. Besides it would be counter-intuitive to assume a fictionalist interpretation for Peano Arithmetic, because fictionalism is a conviction that mathematics taken at first value is false (Ebba Gullbers, 2016).

CONCLUSION

Given that nominalism represents the nihilism of content, the nominalist propositions of number cannot be said to ascribe properties to individuals or objects or to concepts because there is no content in nominalism to ascribe. The focus of mathematical propositions in nominalism, a version of which is formalism, will be vacuous logical truths since nominalism rejects mathematical existence statements. The strict emphasis on the axiomatic method has rendered the epistemic status of formalistic theories non-referential but strictly formal. Formal consistency, which is one of the defining features of nominalist philosophy of mathematics, is the satisfaction of the coherence theoretic pre-conditions of truth. Coherence confers priority and necessity on the nominalist mathematical propositions. The logical structure of mathematical statements in nominalism is analytic-vacuous.

But propositions in mathematics, though analytic, are not vacuous. The use of bound variables in mathematics negates any vacuous characterization of its propositions. Hence, it would be inconsistent with mathematical practice to assume that propositions of arithmetic do not refer. Although the fictionalist strand of formalism has accepted some level of fictionalist objects in mathematics, it is contended that those allowances are due to Quine's indispensability thesis. Following Quine, Field believes that there is no distinction between analytic and synthetic propositions. Consequently, mathematics is part of the web that is confirmed by physical theories. Field therefore dispenses not only with the entities in mathematics but also with mathematics itself.

The attendant nihilism cannot offer itself as an interpretation for the realist structuralism of Peano arithmetic, which, though it did not commit to any unique ontology, makes such a commitment a necessary hypothetical requirement for its satisfaction due to the valid use of bound variables.

REFERENCES

1. Achebe, C. (1980). *Arrow of God.* African Writers Series 6. London: Heineman.
2. Adams, M. M. (1977). "Ockham's Nominalism and Real Entities". *The Philosophical Review.* 86(2), 144 – 170. *JSTOR.* Web. 19 April, 2022.
3. Benacerraf, P & Putnam, H. (1985). *Philosophy of Mathematics: Selected Readings*. 2nd ed. Cambridge: Cambridge University Press. Web. 19 March, 2022.
4. Burgess, J. (2004). "Mathematics and Bleak House". *Philosophia Mathematica*. 12(3), 18 – 36. Web. 19 March, 2022.
5. Ebba, G. (2011). *Objective and Objectivity Alternative to Mathematical Realism.* Umea: Print and Media. Web. 21 March, 2022.
6. Finedictionary. "Conceptualism". *Dex Form.* n.d, n.pag. Web. 19 March, 2022.
7. Gödel, K. (1986). *Collected Works Volume 1*. Ed. Solomon Feferman: Oxford: Oxford University Press.
8. Hamilton, A. G. (1978). *Logic for Mathematics.* Cambridge: Cambridge University Press.
9. Hoffman, S. (2004). "Kitcher, Ideal Agent, and Fictionalism". *Philosophia Mathematica*. 12(3) 3 – 17. Web. 19 March, 2022.
10. Kant, I. (1965). *Critique of Pure Reason*. New York: St. Martin's Press.
11. Korner, S. (1971). *The Philosophy of Mathematics: An Introduction.* London: Hutchinson and Company.
12. Legg, C. (2011). "Predication and the Problem of Universals". *Philosophical Papers.* 30(2) 117 – 143. Web. 19 March, 2022.
13. Peano, G. (1967). "The Principles of Arithmetic Presented by a New Method". *From Frege to Gödel: A Source Book in Mathematical Logic.* Ed. Jean van Heijenoort. Cambridge: Harvard University Press, 82 – 114.
14. Quine, V.O. (1971). *From a Logical Point of View*. London: Oxford University Press.
15. Raatikainen, P. (2003). "Hilbert's Program Revisited." *Synthese*. 137: 157- 177. Web. 5 April, 2022.
16. Raatikainen, P. (2007). "On the Philosophical Relevance of Gödel's Incompleteness Theorems". *Gödel's Incompleteness Theorem.* N.p Web 5th April, 2022.
17. Resnik, M. (1980). *Frege and the Philosophy of Mathematics*. London: Cornet University Press.
18. Russell, B. (1998). *Introduction to Mathematical Philosophy.* London: Rutledge.
19. Russell, B. (1992). *The Principles of Mathematics*. London: Rutledge.

20. Shapiro, S. (2005). "Philosophy of Mathematics and its Logic". *The Oxford Handbook of Philosophy of Mathematics and Logic*. n.d, 6 – 34. Web. Draft. 17 May, 2022.