STRESS- STRAIN STATE AND DESTRUCTION OF AN ELASTIC HOMOGENEOUS ROUND PLATE OF VARIABLE THICKNESS

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Annotation

In this article considers a plane problem of elasticity theory for a round plate of variable thickness.
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Let there be an elastic homogeneous round plate of variable thickness. Let's take the origin of coordinates in the center of the circle, the radius of which is denoted by R. The Cartesian coordinates x, y in the median plane are the plane of symmetry. A round plate of variable thickness is in a generalized plane-stressed state. We suppose, that the thickness of the plate 2h(x,y) satisfies the following conditions:

$$0 < h_1 \leq h(x, y) \leq h_2,$$

where h2 and h1, respectively, are the largest and smallest values of the thickness of the round plate. The function of plate thickness   can be represented as follows:

$$h(x, y) = h_0 \left[1 + \varepsilon \bar{h}(x, y)\right] ,(1)$$

where $h_0 = (h_1 + h_2)/2$; $\varepsilon = (h_2 - h_1)/(h_1 + h_2)$- is a small parameter; $\bar{h}(x, y)$ –s some known dimensionless continuous function $\left(-1 \leq \bar{h}(x, y) \leq 1\right)$.

With a given law of change in the thickness of the round plate, the small parameter ε will be constant.

Let external stresses acting on the bypass of the contour L of a round plate on $\sigma_r - i\tau_{r\theta} = f_1(\theta) - if_2(\theta)$ on the L, (2)

however , the main vector and the main moment of these forces are zero.

The problem is to find the stress-strain state of a round plate of variable thickness under external force action (2).

Let write down the equations of static deformation of a round plate in the following form

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0; \qquad \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0,$$

Hooke's Law:

$$N_x = \frac{2Eh}{1-\mu^2}\left(\frac{\partial u}{\partial x} + \mu\frac{\partial \upsilon}{\partial y}\right); \qquad N_y = \frac{2Eh}{1-\mu^2}\left(\frac{\partial \upsilon}{\partial y} + \mu\frac{\partial u}{\partial x}\right); \qquad (4)$$

$$N_{xy} = \frac{Eh}{1+\mu}\left(\frac{\partial u}{\partial y} + \frac{\partial \upsilon}{\partial x}\right),$$

where N_x, N_y, N_{xy} are, respectively, the normal and shear forces per unit length; u, υ are the components of the displacement vector; E is the elastic modulus of the plate material; μ is the Poisson's ratio of the plate material.

To solve the system of equations of static deformation of a round plate of variable thickness is used of the method perturbation

$$N_x = N_x^{(0)} + \varepsilon N_x^{(1)} + \ldots; \qquad N_y = N_y^{(0)} + \varepsilon N_y^{(1)} + \ldots; \qquad (5)$$

$$N_{xy} = N_{xy}^{(0)} + \varepsilon N_{xy}^{(1)} + \ldots; \qquad u = u_0 + \varepsilon u_1 + \ldots; \qquad \upsilon = \upsilon_0 + \varepsilon \upsilon_1 + \ldots.$$

Using the procedure of the perturbation method, we obtain the equations for each approximation. In the obtained equations, the equations of the zero approximation coincide with the equations of the classical plane problem of elasticity theory, and the equations of the first approximation are the equation of the plane problem of elasticity theory with a volumetric force determined according to [1]

$$X_1 = N_x^{(0)}\frac{\partial \overline{h}}{\partial x} + N_{xy}^{(0)}\frac{\partial \overline{h}}{\partial y}; \qquad Y_1 = N_y^{(0)}\frac{\partial \overline{h}}{\partial y} + N_{xy}^{(0)}\frac{\partial \overline{h}}{\partial x} \qquad (6)$$

The components X_2, Y_2 , of the volumetric force for the second and subsequent approximations are determined similarly.

The boundary conditions of the problem according to the perturbation method have the following form:

for the zero approximation :

$$N_r^{(0)} = f_1^*(\theta); \qquad N_{r\theta}^{(0)} = f_2^*(\theta) \qquad \text{at } |z| = R \qquad (7)$$

for the first approximation:

$$N_r^* = 0; \qquad N_{r\theta}^* = 0 \qquad \text{при } |z| = R \qquad (8)$$

Note, that when deriving the equations of the first approximation, we the following notation are adopted

$$N_r^* = N_r^{(1)} - N_{r_0}^{(1)}; \qquad N_{r_0}^{(1)} = \overline{h}(x,y)N_r^{(0)};$$

$$N_\theta^* = N_\theta^{(1)} - N_{\theta_0}^{(1)}; \qquad N_{\theta_0}^{(1)} = \overline{h}(x,y)N_\theta^{(0)}; \qquad (9)$$

$$N_{r\theta}^* = N_{r\theta}^{(1)} - N_{r\theta_0}^{(1)}; \qquad N_{r\theta_0}^{(1)} = \overline{h}(x,y)N_{r\theta}^{(0)}$$

The solution of problem (7) for the zero approximation is known [2].

Let us pass to the solution of problem (8) of the first approximation. In the presence of volumetric forces , the solution of the first approximation is represented as a sum

$$N_r^* = N_{r_*}^{(1)} + N_{r_1}^{(1)}; \qquad N_\theta^* = N_{\theta_*}^{(1)} + N_{\theta_1}^{(1)}; \qquad N_{r\theta}^* = N_{r\theta_*}^{(1)} + N_{r\theta_1}^{(1)} \qquad (10)$$

Here $N_{r_*}^{(1)}$, $N_{\theta_*}^{(1)}$, $N_{r\theta_*}^{(1)}$ - is a partial solution of the equations of the plane theory of elasticity in the presence of a volumetric force determined by formulas (6); $N_{r_1}^{(1)}$, $N_{\theta_1}^{(1)}$, $N_{r\theta_1}^{(1)}$ - is a general solution of the equations of the plane theory of elasticity in the absence of volumetric forces.

Using the method of A.G. Ugodchikov [3], for efforts , in the first approximation we obtain the following general representations

$$\frac{N_r^* + N_\theta^*}{2h_0} = 4\operatorname{Re}\left[\Phi_1(z) - \frac{1}{2(1+\kappa_0)} \cdot \frac{\partial F_1}{\partial z} \right] \qquad (11)$$

$$\frac{N_\theta^* - N_r^* + 2iN_{r\theta}^*}{2h_0} = 2\left[\overline{z}\Phi_1(z) + \Psi_1(z) + \frac{1}{2(1+\kappa_0)}\frac{\partial}{\partial z}\left(\kappa_0 \overline{F_1} - \overline{Q_1} \right) \right]e^{2i\theta},$$

Where $\kappa_0 = (3-\mu)/(1+\mu)$ – is Muskhelishvili constant.

These general representations (11) include two analytical functions and a complex variable and two functions and, representing any particular solutions of the following differential equations: $\dfrac{\partial^2 F_1}{\partial z \partial \overline{z}} = F$;

$$\frac{\partial^2 Q_1}{\partial z^2} = \overline{F} , \qquad (12)$$

Where $F = X_1 + iY_1 = \dfrac{\partial \overline{h}}{\partial x}\left(N_x^{(0)} + iN_{xy}^{(0)} \right) + i\dfrac{\partial \overline{h}}{\partial y}\left(N_y^{(0)} - iN_{xy}^{(0)} \right) \qquad (13)$

Using the relations (11), we find

$$N_r^* - iN_{r\theta}^* = \Phi_1(z) + \overline{\Phi_1(z)} - e^{2i\theta}\left[\overline{z}\Phi_1'(z) + \Psi_1(z) \right] - \qquad (14)$$

$$-\frac{1}{1+\kappa_0}\operatorname{Re}\frac{\partial F_1}{\partial z} - \frac{1}{2(1+\kappa_0)}\left[\frac{\partial}{\partial z}\left(\kappa_0 \overline{F_1} - \overline{Q_1} \right) \right]e^{2i\theta}$$

To find complex potentials , we come to the following boundary value problem

$$\Phi_1(z) + \overline{\Phi_1(z)} - e^{2i\theta}\left[\overline{z}\Phi_1'(z) + \Psi_1(z)\right] = f(\theta) \quad \text{at} \ |z| = R \tag{15}$$

Here $f(\theta) = \dfrac{1}{1+\kappa_0}\operatorname{Re}\dfrac{\partial F_1}{\partial z} + \dfrac{1}{2(1+\kappa_0)}\left[\dfrac{\partial}{\partial z}\left(\kappa_0 \overline{F_1} - \overline{Q_1}\right)\right]e^{2i\theta}$ at $z = Re^{i\theta}$

The functions $F_1(z,\overline{z})$ and $Q_1(z,\overline{z})$ can be formally written in the following form:

$$F_1(z,\overline{z}) = \int\limits^{z} dz \int\limits^{\overline{z}} F(z,\overline{z})d\overline{z} \ ; \ Q_1(z,\overline{z}) = \int\limits^{z} dz \int\limits^{z} \overline{F(z,\overline{z})}dz \tag{16}$$

The stress state in the first approximation is determined using two analytical functions $\Phi_1(z)$ and . $\Psi_1(z)$. Complex potentials $\Phi_1(z)$ and $\Psi_1(z)$ are determined from the boundary condition (15) by the method of N.I. Muskhelishvili [2]

$$\Phi_1(z) = \dfrac{1}{2\pi i}\int\limits_L f(\tau)\left(\dfrac{1}{\tau - z} - \dfrac{1}{2\tau}\right)d\tau \ ; \tag{17}$$

$$\Psi_1(z) = \dfrac{1}{z^2}\Phi_1(z) + \dfrac{1}{z^2}\overline{\Phi}_1\left(\dfrac{1}{z}\right) - \dfrac{1}{z}\Phi_1'(z)$$

According to this method , the solution of the problem of elasticity theory for a round plate of variable thickness can be written as:

$$N_r = \left[1 + \varepsilon\overline{h}(r,\theta)\right]N_r^{(0)} + \varepsilon N_r^* \ ;$$

$$N_\theta = \left[1 + \varepsilon\overline{h}(r,\theta)\right]N_\theta^{(0)} + \varepsilon N_\theta^* , \tag{18}$$

$$N_{r\theta} = \left[1 + \varepsilon\overline{h}(r,\theta)\right]N_{r\theta}^{(0)} + \varepsilon N_{r\theta}^*$$

Here efforts N_r^*, N_θ^*, $N_{r\theta}^*$ are determined using relations (11). The obtained formulas (18) allow us to calculate the effect of the variability of the thickness of a round plate on the stress distribution.

Let's consider particular examples.

1. A disk under the action of concentrated forces applied to the contour:

Let two equal and opposite forces (p, 0) and (– p, 0) act on the contour of the disk (p, 0) и (– p, 0) parallel to the axis of the abscissa and applied at the points $z_1 = Re^{i\theta}$ и $z_2 = Re^{i(\pi-\alpha)} = -Re^{-i\alpha}$.

The solution of the zero approximation in this case has [2] of the form:

$$\dfrac{N_x^{(0)}}{2h_0} = \dfrac{2p}{\pi}\left(\dfrac{\cos^3\theta_1}{r_1} + \dfrac{\cos^3\theta_2}{r_2}\right) - \dfrac{p}{\pi R}\cos\alpha \ ;$$

$$\dfrac{N_y^{(0)}}{2h_0} = \dfrac{2p}{\pi}\left(\dfrac{\sin^2\theta_1\cos\theta_1}{r_1} + \dfrac{\sin^2\theta_2\cos\theta_2}{r_2}\right) - \dfrac{p}{\pi R}\cos\alpha \ ; \tag{19}$$

$$\dfrac{N_{xy}^{(0)}}{2h_0} = -\dfrac{2p}{\pi}\left\{\dfrac{\sin\theta_1\cos^2\theta_1}{r_1} - \dfrac{\sin\theta_2\cos^2\theta_2}{r_2}\right\},$$

where $z_1 - z = r_1 e^{-i\theta_1} \ ; \qquad z_2 - z = r_2 e^{-i\theta_2}$.

To compose the volumetric force in the first approximation, we have (13). Using the relations (16) after integration, we find the functions $F_1(z, \bar{z})$ and $Q_1(z, \bar{z})$. By the found functions $F_1(z, \bar{z})$ and $Q_1(z, \bar{z})$ according to (15) we find the function $f(\theta)$ After determining the complex potential $\Phi_1(z)$ and $\Psi_1(z)$ it is possible to investigate the stress-strain state of around disk of variable thickness

2. Rotating disk with attached concentrated masses:

3. Let an elastic thin disk of variable thickness rotate with angular velocity $\square$ around its center and let concentrated masses be attached to it (at arbitrary points).

The solution of the problem in the zero approximation has [2] the form

$$\frac{N_x^{(0)}}{2h_0} = -\frac{3+\mu}{8}\rho^2\omega^2 R^2 - \frac{(1+\mu)}{4}\rho\omega^2 r^2 - \frac{1-\mu}{8}\rho^2\omega^2 r^2\cos 2\theta +$$

$$+\frac{m\ell\omega^2}{h_0\pi}\left\{\frac{\cos^3\theta_1}{r_1}+\frac{\cos^3\theta_2}{r_2}\right\}-\frac{m\ell\omega^2}{2h_0\pi R}\cos\alpha\,; \qquad (20)$$

$$\frac{N_x^{(0)}}{2h_0} = \frac{1-\mu}{8}\rho\omega^2 r^2\cos 2\theta + \frac{3+\mu}{8}\rho\omega^2 R^2 - \frac{1+\mu}{4}\rho\omega^2 r^2 +$$

$$+\frac{m\ell\omega^2}{h_0\pi}\left\{\frac{\sin^2\theta_1\cos\theta_1}{r_1}+\frac{\sin^2\theta_2\cos\theta_2}{r_2}\right\}-\frac{m\ell\omega^2}{2h_0\pi R}\cos\alpha\,;$$

$$\frac{N_{x\text{и}}^{(0)}}{2h_0} = -\frac{1-\mu}{8}\rho\omega^2 r^2\sin 2\theta - \frac{m\ell\omega^2}{h_0\pi}\left\{\frac{\sin\theta_1\cos^2\theta_1}{r_1}-\frac{\sin\theta_2\cos^2\theta_2}{r_2}\right\}$$

Here m - is the attached mass at the points z_1 and z_2; ℓ is the distance from the point of attachment of the load to the axis of rotation, the remaining designations are preserved. Using the ralations (13) and (20), we find the components of the volumetric force.

After integrating equations (16), we find the functions $F_1(z, \bar{z})$ and $Q_1(z, \bar{z})$. Using the found functions $F_1(z, \bar{z})$ and $Q_1(z, \bar{z})$ according to (15) we define the function $f(\theta)$. Integrating (17), we find the complex potentials $\Phi_1(z)$ и $\Psi_1(z)$. Knowing the complex potentials $\Phi_1(z)$ and $\Psi_1(z)$, using the relations (11), we find N_r^*, N_θ^* and. $N_{r\theta}^*$. The stress state of the rotating disk is investigated according to the relations (18).

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