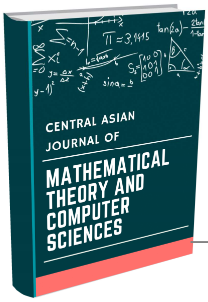

CENTRAL ASIAN JOURNAL OF MATHEMATICAL THEORY AND COMPUTER SCIENCES

http://cajmtcs.centralasianstudies.org/index.php/CAJMTCS

Volume: 03 Issue: 05 | May 2022 ISSN: 2660-5309

Transportation Problem in Operations Research and its Applications

Krisn Pratap Meena

Assistant Professor, Department of Mathematics, S.R.R.M. Govt. College, Nawalgarh, Rajasthan, India

Annotation:

In mathematics and economics, transportation theory or transport theory is a name given to the study of optimal transportation and allocation of resources. The problem was formalized by the French mathematician Gaspard Monge in 1781.[1]

In the 1920s A.N. Tolstoi was one of the first to study the transportation problem mathematically. In 1930, in the collection Transportation Planning Volume I for the National Commissariat of Transportation of the Soviet Union, he published a paper "Methods of Finding the Minimal Kilometrage in Cargo-transportation in space".[2][3]

Major advances were made in the field during World War II by the Soviet mathematician and economist Leonid Kantorovich.[4] Consequently, the problem as it is stated is sometimes known as the Monge–Kantorovich transportation problem.[5] The linear programming formulation of the transportation problem is also known as the Hitchcock–Koopmans transportation problem.[6]

ARTICLEINFO

Article history:
Received 18 Mar 2022
Revised form 15 Apr 2022
Accepted 31 May 2022

Key words: transportation, operations, research, applications, mathematical, economics.

---***---

INTRODUCTION

Motivation

Mines and factories

Suppose that we have a collection of m mines mining iron ore, and a collection of n factories which use the iron ore that the mines produce. Suppose for the sake of argument that these mines and factories form two disjoint subsets M and F of the Euclidean plane R^2. Suppose also that we have a cost function $c : R^2 \times R^2 \rightarrow [0, \infty)$, so that c(x, y) is the cost of transporting one shipment of iron from x to y. For simplicity, we ignore the time taken to do the transporting. We also assume that each mine can supply only one factory (no splitting of shipments) and that each factory requires precisely one shipment to be in operation (factories cannot work at half- or double-capacity). Having made the above assumptions, a transport plan is a bijection $T : M \rightarrow F$.[1,2,3]

Moving books: the importance of the cost function

The following simple example illustrates the importance of the cost function in determining the optimal transport plan. Suppose that we have n books of equal width on a shelf (the real line), arranged in a single contiguous block. We wish to rearrange them into another contiguous block, but shifted one book-width to the right. Two obvious candidates for the optimal transport plan present themselves:

1. move all n books one book-width to the right ("many small moves");
2. move the left-most book n book-widths to the right and leave all other books fixed ("one big move").

If the cost function is proportional to Euclidean distance ($c(x, y) = \alpha|x - y|$) then these two candidates are both optimal. If, on the other hand, we choose the strictly convex cost function proportional to the square of Euclidean distance ($c(x, y) = \alpha|x - y|^2$), then the "many small moves" option becomes the unique minimizer.

Note that the above cost functions consider only the horizontal distance traveled by the books, not the horizontal distance traveled by a device used to pick each book up and move the book into position. If the latter is considered instead, then, of the two transport plans, the second is always optimal for the Euclidean distance, while, provided there are at least 3 books, the first transport plan is optimal for the squared Euclidean distance.[4,5,6]

Applications

The Monge–Kantorovich optimal transport has found applications in wide range in different fields. Among them are:

- Image registration and warping[17]
- Reflector design[18]
- Retrieving information from shadowgraphy and proton radiography[19]
- Seismic tomography and reflection seismology[20]
- The broad class of economic modelling that involves gross substitutes property (among others, models of matching and discrete choice).

DISCUSSION

Operations research (British English: operational research) (U.S. Air Force Specialty Code: Operations Analysis), often shortened to the initialism OR, is a discipline that deals with the development and application of analytical methods to improve decision-making.[1] The term management science is occasionally used as a synonym.[2]

Employing techniques from other mathematical sciences, such as modeling, statistics, and optimization, operations research arrives at optimal or near-optimal solutions to decision-making problems. Because of its emphasis on practical applications, operations research has overlapped with many other disciplines, notably industrial engineering. Operations research is often concerned with determining the extreme values of some real-world objective: the maximum (of profit, performance, or yield) or minimum (of loss, risk, or cost). Originating in military efforts before World War II, its techniques have grown to concern problems in a variety of industries.[3]

Operational research (OR) encompasses the development and the use of a wide range of problem-solving techniques and methods applied in the pursuit of improved decision-making and efficiency, such as simulation, mathematical optimization, queueing theory and other stochastic-process models, Markov decision processes, econometric methods, data envelopment analysis, ordinal priority approach, neural networks, expert systems, decision analysis, and the analytic hierarchy process.[4] Nearly all of these techniques involve the construction of mathematical models that attempt to describe the system. Because of the computational and statistical nature of most of these fields, OR also has strong ties to computer science and analytics. Operational researchers faced with a new problem must determine which of these techniques are most appropriate given the nature of the system, the goals for improvement, and constraints on time and computing power, or develop a new technique specific to the problem at hand (and, afterwards, to that type of problem).[7,8,9]

The major sub-disciplines in modern operational research, as identified by the journal Operations Research,[5] are:

- Computing and information technologies
- Financial engineering
- Manufacturing, service sciences, and supply chain management
- Policy modeling and public sector work
- Revenue management
- Simulation
- Stochastic models
- Transportation theory (mathematics)
- Game theory for strategies
- Linear programming
- Nonlinear programming
- Integer programming in NP-complete problem specially for 0-1 integer linear programming for binary
- Dynamic programming in Aerospace engineering and Economics
- Information theory used in Cryptography, Quantum computing
- Quadratic programming for solutions of Quadratic equation and Quadratic function

History

In the decades after the two world wars, the tools of operations research were more widely applied to problems in business, industry, and society. Since that time, operational research has expanded into a field widely used in industries ranging from petrochemicals to airlines, finance, logistics, and government, moving to a focus on the development of mathematical models that can be used to analyse and optimize sometimes complex systems, and has become an area of active academic and industrial research.[3]

Historical origins

In the 17th century, mathematicians Blaise Pascal and Christiaan Huygens solved problems involving sometimes complex decisions (problem of points) by using game-theoretic ideas and expected values; others, such as Pierre de Fermat and Jacob Bernoulli, solved these types of problems using combinatorial reasoning instead.[6] Charles Babbage's research into the cost of transportation and sorting of mail led to England's universal "Penny Post" in 1840, and to studies into the dynamical behaviour of railway vehicles in defence of the GWR's broad gauge.[7] Beginning in the 20th century, study of inventory management could be considered[by whom?] the origin of modern operations research with economic order quantity developed by Ford W. Harris in 1913. Operational research mayhave originated in the efforts of military planners during World War I (convoy theory and Lanchester's laws). Percy Bridgman brought operational research to bear on problems in physics in the 1920s and would later attempt to extend these to the social sciences.[8]

Modern operational research originated at the Bawdsey Research Station in the UK in 1937 as the result of an initiative of the station's superintendent, A. P. Rowe and Robert Watson-Watt.[9] Rowe conceived the idea as a means to analyse and improve the working of the UK's early-warning radar system, code-named "Chain Home" (CH). Initially, Rowe analysed the operating of the radar equipment and its communication networks, expanding later to include the operating personnel's behaviour. This revealed unappreciated limitations of the CH network and allowed remedial action to be taken.[10]

Scientists in the United Kingdom (including Patrick Blackett (later Lord Blackett OM PRS), Cecil Gordon, Solly Zuckerman, (later Baron Zuckerman OM, KCB, FRS), C. H. Waddington, Owen Wansbrough-Jones, Frank Yates, Jacob Bronowski and Freeman Dyson), and in the United States (George Dantzig) looked for ways to make better decisions in such areas as logistics and training schedules.[10,11,12]

Second World War

The modern field of operational research arose during World War II. In the World War II era, operational research was defined as "a scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control".[11] Other names for it included operational analysis (UK Ministry of Defence from 1962)[12] and quantitative management.[13]

During the Second World War close to 1,000 men and women in Britain were engaged in operational research. About 200 operational research scientists worked for the British Army.[14]

Patrick Blackett worked for several different organizations during the war. Early in the war while working for the Royal Aircraft Establishment (RAE) he set up a team known as the "Circus" which helped to reduce the number of anti-aircraft artillery rounds needed to shoot down an enemy aircraft from an average of over 20,000 at the start of the Battle of Britain to 4,000 in 1941.[15]

In 1941, Blackett moved from the RAE to the Navy, after first working with RAF Coastal Command, in 1941 and then early in 1942 to the Admiralty.[16] Blackett's team at Coastal Command's Operational Research Section (CC-ORS) included two future Nobel prize winners and many other people who went on to be pre-eminent in their fields.[17][18] They undertook a number of crucial analyses that aided the war effort. Britain introduced the convoy system to reduce shipping losses, but while the principle of using warships to accompany merchant ships was generally accepted, it was unclear whether it was better for convoys to be small or large. Convoys travel at the speed of the slowest member, so small convoys can travel faster. It was also argued that small convoys would be harder for German U-boats to detect. On the other hand, large convoys could deploy more warships against an attacker. Blackett's staff showed that the losses suffered by convoys depended largely on the number of escort vessels present, rather than the size of the convoy. Their conclusion was that a few large convoys are more defensible than many small ones.[19]

While performing an analysis of the methods used by RAF Coastal Command to hunt and destroy submarines, one of the analysts asked what colour the aircraft were. As most of them were from Bomber Command they were painted black for night-time operations. At the suggestion of CC-ORS a test was run to see if that was the best colour to camouflage the aircraft for daytime operations in the grey North Atlantic skies. Tests showed that aircraft painted white were on average not spotted until they were 20% closer than those painted black. This change indicated that 30% more submarines would be attacked and sunk for the same number of sightings.[20] As a result of these findings Coastal Command changed their aircraft to using white undersurfaces.[13,14,15]

Other work by the CC-ORS indicated that on average if the trigger depth of aerial-delivered depth charges were changed from 100 to 25 feet, the kill ratios would go up. The reason was that if a U-boat saw an aircraft only shortly before it arrived over the target then at 100 feet the charges would do no damage (because the U-boat wouldn't have had time to descend as far as 100 feet), and if it saw the aircraft a long way from the target it had time to alter course under water so the chances of it being within the 20-foot kill zone of the charges was small. It was more efficient to attack those submarines close to the surface when the targets' locations were better known than to attempt their destruction at greater depths when their positions could only be guessed. Before the change of settings from 100 to 25 feet, 1% of submerged U-boats were sunk and 14% damaged. After the change, 7% were sunk and 11% damaged; if submarines were caught on the surface but had time to submerge just before being attacked, the numbers rose to 11% sunk and 15% damaged. Blackett observed "there can be few cases where such a great operational gain had been obtained by such a small and simple change of tactics".[21]

Bomber Command's Operational Research Section (BC-ORS), analyzed a report of a survey carried out by RAF Bomber Command. For the survey, Bomber Command inspected all bombers returning from bombing raids over Germany over a particular period. All damage inflicted by German air defenses was noted and the recommendation was given that armor be added in the most heavily damaged areas. This recommendation was not adopted because the fact that the aircraft were able to return with these areas damaged indicated the areas were not vital, and adding armor to non-vital areas where damage is acceptable reduces aircraft performance. Their suggestion to remove some of the crew so that an aircraft loss would result in fewer personnel losses, was also rejected by RAF command. Blackett's team made the logical recommendation that the armor be placed in the areas which were completely untouched by damage in the bombers who returned. They reasoned that the survey was biased, since it only included aircraft that returned to Britain. The areas untouched in returning aircraft were probably vital areas, which, if hit, would result in the loss of the aircraft.[22] This story has been disputed,[23] with a similar damage assessment study completed in the US by the Statistical Research Group at Columbia University,[24] the result of work done by Abraham Wald.[25]

When Germany organized its air defences into the Kammhuber Line, it was realized by the British that if the RAF bombers were to fly in a bomber stream they could overwhelm the night fighters who flew in individual cells directed to their targets by ground controllers. It was then a matter of calculating the statistical loss from collisions against the statistical loss from night fighters to calculate how close the bombers should fly to minimize RAF losses.[26]

The "exchange rate" ratio of output to input was a characteristic feature of operational research. By comparing the number of flying hours put in by Allied aircraft to the number of U-boat sightings in a given area, it was possible to redistribute aircraft to more productive patrol areas. Comparison of exchange rates established "effectiveness ratios" useful in planning. The ratio of 60 mines laid per ship sunk was common to several campaigns: German mines in British ports, British mines on German routes, and United States mines in Japanese routes.[27]

Operational research doubled the on-target bomb rate of B-29s bombing Japan from the Marianas Islands by increasing the training ratio from 4 to 10 percent of flying hours; revealed that wolf-packs of three United States submarines were the most effective number to enable all members of the pack to engage targets discovered on their individual patrol stations; revealed that glossy enamel paint was more effective camouflage for night fighters than conventional dull camouflage paint finish, and a smooth paint finish increased airspeed by reducing skin friction.[27]

On land, the operational research sections of the Army Operational Research Group (AORG) of the Ministry of Supply (MoS) were landed in Normandy in 1944, and they followed British forces in the advance across Europe. They analyzed, among other topics, the effectiveness of artillery, aerial bombing and anti-tank shooting.

In 1947 under the auspices of the British Association, a symposium was organized in Dundee. In his opening address, Watson-Watt offered a definition of the aims of OR:

"To examine quantitatively whether the user organization is getting from the operation of its equipment the best attainable contribution to its overall objective."[9]

With expanded techniques and growing awareness of the field at the close of the war, operational research was no longer limited to only operational, but was extended to encompass equipment procurement, training, logistics and infrastructure. Operations research also grew in many areas other than the military once scientists learned to apply its principles to the civilian sector. The development of the simplex algorithm for linear programming was in 1947.[28]

In the 1950s, the term Operations Research was used to describe heterogeneous mathematical methods such as game theory, dynamic programming, linear programming, warehousing, spare parts theory, queue theory, simulation and production control, which were used primarily in civilian industry. Scientific societies and journals on the subject of operations research were founded in the 1950s, such as the Operation Research Society of America (ORSA) in 1952 and the Institute for Management Science (TIMS) in 1953.[29] Philip Morse, the head of the Weapons Systems Evalution Group of the Pentagon, became the first president of ORSA and attracted the companies of the military-industrial complex to ORSA, which soon had more than 500 members. In the 1960s, ORSA reached 8000 members. Consulting companies also founded OR groups. In 1953, Abraham Charnes and William Cooper published the first textbook on Linear Programming.

In the 1950s and 1960s, chairs of operations research were established in the U.S. and United Kingdom (from 1964 in Lancaster) in the management faculties of universities. Further influences from the U.S. on the development of operations research in Western Europe can be traced here. The authoritativeOR textbooks from the U.S. were published in Germany in German language and in France in French (but not in Italiansuch as the book by George Dantzig "Linear Programming"(1963) and the book by C. West Churchman et al. "Introduction to Operations Research"(1957). The latter was also published in Spanish in 1973, opening at the same time Latin American readers to Operations Research. NATO gave important impulses for the spread of Operations Research in Western Europe; NATO headquarters (SHAPE) organised four conferences on OR in the 1950s – the one in 1956 with 120 participants – bringing OR to mainland Europe. Within NATO, OR was also known as "Scientific Advisory" (SA) and was grouped together in the Advisory Group of Aeronautical Research and Development (AGARD). SHAPE and AGARD organized an OR conference in April 1957 in Paris. When France withdrew from the NATO military command structure, the transfer of NATO headquarters from France to Belgium led to the institutionalization of OR in Belgium, where Jacques Drèze founded CORE, the Center for Operations Research and Econometrics at the Catholic University of Leuven in 1966.

With the development of computers over the next three decades, Operations Research can now solve problems with hundreds of thousands of variables and constraints. Moreover, the large volumes of data required for such problems can be stored and manipulated very efficiently."[28] Much of operations research (modernly known as 'analytics') relies upon stochastic variables and a therefore access to truly random numbers. Fortunately, the cybernetics field also required the same level of randomness. The development of increasingly better random number generators has been a boon to both disciplines. Modern applications of operations research includes city planning, football strategies, emergency planning, optimizing all facets of industry and economy, and undoubtedly with the likelihood of the inclusion of terrorist attack planning and definitely counterterrorist attack planning. More recently, the research approach of operations research, which dates back to the 1950s, has been criticized for being collections of mathematical models but lacking an empirical basis of data collection for applications. How to collect data is not presented in the textbooks. Because of the lack of data, there are also no computer applications in the textbooks.[30]

Problems addressed

- Critical path analysis or project planning: identifying those processes in a multiple-dependency project which affect the overall duration of the project
- Floorplanning: designing the layout of equipment in a factory or components on a computer chip to reduce manufacturing time (therefore reducing cost)

- Network optimization: for instance, setup of telecommunications or power system networks to maintain quality of service during outages
- Resource allocation problems
- Facility location
- Assignment Problems:
- Assignment problem
- Generalized assignment problem
- Quadratic assignment problem
- Weapon target assignment problem
- Bayesian search theory: looking for a target
- Optimal search
- Routing, such as determining the routes of buses so that as few buses are needed as possible
- Supply chain management: managing the flow of raw materials and products based on uncertain demand for the finished products
- Project production activities: managing the flow of work activities in a capital project in response to system variability through operations research tools for variability reduction and buffer allocation using a combination of allocation of capacity, inventory and time[31][32]
- Efficient messaging and customer response tactics
- Automation: automating or integrating robotic systems in human-driven operations processes
- Globalization: globalizing operations processes in order to take advantage of cheaper materials, labor, land or other productivity inputs
- Transportation: managing freight transportation and delivery systems (Examples: LTL shipping, intermodal freight transport, travelling salesman problem, driver scheduling problem)
- Scheduling:
- Personnel staffing
- Manufacturing steps
- Project tasks
- Network data traffic: these are known as queueing models or queueing systems.
- Sports events and their television coverage
- Blending of raw materials in oil refineries
- Determining optimal prices, in many retail and B2B settings, within the disciplines of pricing science
- Cutting stock problem: Cutting small items out of bigger ones.

Operational research is also used extensively in government where evidence-based policy is used.

Management science

In 1967 Stafford Beer characterized the field of management science as "the business use of operations research".[33] Like operational research itself, management science (MS) is an interdisciplinary branch of applied mathematics devoted to optimal decision planning, with strong links with economics, business, engineering, and other sciences. It uses various scientific research-based principles, strategies, and analytical methods including mathematical modeling, statistics and numerical algorithms to improve an organization's ability to enact rational and meaningful management decisions by arriving at optimal or near-optimal solutions to sometimes complex decision problems. Management scientists help businesses to achieve their goals using the scientific methods of operational research.[16]

The management scientist's mandate is to use rational, systematic, science-based techniques to inform and improve decisions of all kinds. Of course, the techniques of management science are not restricted to business applications but may be applied to military, medical, public administration, charitable groups, political groups or community groups.

Management science is concerned with developing and applying models and concepts that may prove useful in helping to illuminate management issues and solve managerial problems, as well as designing and developing new and better models of organizational excellence.[34]

The application of these models within the corporate sector became known as management science.[35]

Applications

Applications are abundant such as in airlines, manufacturing companies, service organizations, military branches, and government. The range of problems and issues to which it has contributed insights and solutions is vast. It includes:[34]

- Scheduling (of airlines, trains, buses etc.)
- Assignment (assigning crew to flights, trains or buses; employees to projects; commitment and dispatch of power generation facilities)
- Facility location (deciding most appropriate location for new facilities such as warehouses; factories or fire station)
- Hydraulics & Piping Engineering (managing flow of water from reservoirs)
- Health Services (information and supply chain management)
- Game Theory (identifying, understanding; developing strategies adopted by companies)
- Urban Design
- Computer Network Engineering (packet routing; timing; analysis)
- Telecom & Data Communication Engineering (packet routing; timing; analysis) [37]

Management is also concerned with so-called soft-operational analysis which concerns methods for strategic planning, strategic decision support, problem structuring methods. In dealing with these sorts of challenges, mathematical modeling and simulation may not be appropriate or may not suffice. Therefore, during the past 30 years a number of non-quantified modeling methods have been developed. [12,13,14]These include:

- stakeholder based approaches including metagame analysis and drama theory
- morphological analysis and various forms of influence diagrams
- cognitive mapping
- strategic choice
- robustness analysis

RESULTS

A transportation problem is a Linear Programming Problem that deals with identifying an optimal solution for transportation and allocating resources to various destinations and from one site to another while keeping the expenditure to a minimum.[18,19,20]

In simple words, the main objective of the Transportation problem is to deliver (from the source to the destination) the resources at the minimum cost.

- Also referred to as Hitchcock Problem.
- It involves transporting a single product from ‘m’ source (origin) to ‘n’ destinations.
- Assumptions: The supply level of each source and the demand at each destination are known.
- Objective: To minimize the total Transportation Cost.

Formulation of Transportation Problem

Let you are supplying the resources from m source (S_i) to n destinations (D_j) such that:

a_i: the quantity available at the source S_i

b_j: the quantity required at the destination D_j

c_{ij}: cost of transportation of one unit resource from S_i to D_j

x_{ij}: units of resources transported from S_i to D_j

$1 <= i <= m, 1 <= j <= n$

So, the Total Cost of Transposition is:

$(c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + \ldots\ldots + c_{1n}x_{1n}) + (c_{21}x_{21} + c_{22}x_{22} + c_{23}x_{23} + \ldots\ldots + c_{2n}x_{2n}) + \ldots\ldots. + (c_{m1}x_{m1} + c_{m2}x_{m2} + c_{m3}x_{m3} + \ldots\ldots + c_{mn}x_{mn})$

As we already mentioned, our objective is to minimize the Total Cost:

$\text{Min } Z = (c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + \ldots\ldots + c_{1n}x_{1n}) + (c_{21}x_{21} + c_{22}x_{22} + c_{23}x_{23} + \ldots\ldots + c_{2n}x_{2n}) + \ldots\ldots. + (c_{m1}x_{m1} + c_{m2}x_{m2} + c_{m3}x_{m3} + \ldots\ldots + c_{mn}x_{mn})$

Subject to:

$x_{i1} + x_{i2} + \ldots\ldots. + x_{in} = a_i \ \& \ x_{1j} + x_{2j} + \ldots\ldots. + x_{mn} = b_j$

$x_{ij} >= 0$,

$i = 1, 2, 3, \ldots\ldots, m, j = 1, 2, 3, \ldots\ldots., n$

The below-given matrix can also represent the above diagram.

Types of Transportation Problems

Transportation problems are broadly classified into balanced and unbalanced, depending on the source’s supply and the requirement at the destination.

Balanced Transportation Problem

Unbalanced Transportation Problem

Example – 1: Check which types of Transportation Problem it is.

Answer – 1: From the above, we have

Total Supply = 5 + 8 + 7 + 14 = 34

Total Demand = 7 + 9 + 18 = 34

Hence, Total Supply = Total Demand

Therefore, it is a Balanced Transportation Problem.

Example – 2: Check whether the given problem is Balanced or Unbalanced.

	D1	D2	D3	Availability
S1	4	3	2	10
S2	2	5	0	13
S3	3	8	6	12
Required	8	5	4	–

Answer – 2: From the above matrix, we have:

Total Supply = 10 + 13 + 12 = 35

Total Demand = 8 + 5 + 4 = 17

Hence, Total Supply != Total Demand

therefore, the given transportation problem is Unbalanced Transportation Problem.

Now, let's see what is the difference is between transportation problem and assignment problem.

Transportation Problem vs. Assignment Problem

Transportation Problem	Assignment Problem
It is used to optimize the transportation cost.	It is about assigning finite source to finite destination (one source is alloted to one destination).
Number of Source and demand may or may not be equal.	Number of source and number of destination must be equal.
If demand and supply are not equal, then transportation problem is known as Unbalanced Transportation Problem.	If number of rows and number of columns are not equal, then the assignment problem is known as Unbalanced Assignment Problem.
It requires to step to solve: Find Initial Solution using North West, Least Cost or Vogel Approximation Find Optimal Solution using MODI method.	It requires only one step to solve. Hungarian Method is sufficient to find the optimal solutions.

Transportation Problem in operational research is a special kind of linear programming problem, having objective to find the minimum cost of transportation of goods from m source to n destination.

Hope this article, helps you to learn more about transportation problem.[20]

CONCLUSION

Operations research or operational research (OR) is an interdisciplinary branch of mathematics which uses methods like mathematical modeling, statistics, and algorithms to arrive at optimal or good decisions in complex problems which are concerned with optimizing the maxima (profit, faster assembly line, greater crop yield, higher bandwidth, etc) or minima (cost loss, lowering of risk, etc) of some objective function. The eventual intention behind using operations research is to elicit a best possible solution to a problem mathematically, which improves or optimizes the performance of the system.

Optimization aims to find the minimum (or maximum) value of an objective function subject to constraints that represent user preferences and/or limitations imposed by the nature of the question at hand. Research in optimization involves the analysis of such mathematical problems and the design of efficient algorithms for solving them. It is therefore no surprise that optimization, while integral to operations research, has become an indispensable tool in other areas such as statistics, machine learning, computer vision, and computational biology, just to name a few. Optimization technologies are shining examples of how deep mathematical techniques help to provide concrete computational tools for solving a diverse suite of problems.[16,17,18]

REFERENCES

1. G. Monge. Mémoire sur la théorie des déblais et des remblais. Histoire de l'Académie Royale des Sciences de Paris, avec les Mémoires de Mathématique et de Physique pour la même année, pages 666–704, 1781.
2. Schrijver, Alexander, Combinatorial Optimization, Berlin ; New York : Springer, 2003. ISBN 3540443894. Cf. p. 362
3. Ivor Grattan-Guinness, Ivor, Companion encyclopedia of the history and philosophy of the mathematical sciences, Volume 1, JHU Press, 2003. Cf. p.831
4. L. Kantorovich. On the translocation of masses. C.R. (Doklady) Acad. Sci. URSS (N.S.), 37:199–201, 1942.
5. Cédric Villani (2003). Topics in Optimal Transportation. American Mathematical Soc. p. 66. ISBN 978-0-8218-3312-4.
6. Singiresu S. Rao (2009). Engineering Optimization: Theory and Practice (4th ed.). John Wiley & Sons. p. 221. ISBN 978-0-470-18352-6.
7. Frank L. Hitchcock (1941) "The distribution of a product from several sources to numerous localities", MIT Journal of Mathematics and Physics 20:224–230 MR0004469.
8. D. R. Fulkerson (1956) Hitchcock Transportation Problem, RAND corporation.
9. L. R. Ford Jr. & D. R. Fulkerson (1962) § 3.1 in Flows in Networks, page 95, Princeton University Press
10. L. Ambrosio, N. Gigli & G. Savaré. Gradient Flows in Metric Spaces and in the Space of Probability Measures. Lectures in Mathematics ETH Zürich, Birkhäuser Verlag, Basel. (2005)
11. Angenent, S.; Haker, S.; Tannenbaum, A. (2003). "Minimizing flows for the Monge–Kantorovich problem". SIAM J. Math. Anal. 35 (1): 61–97. CiteSeerX 10.1.1.424.1064. doi:10.1137/S0036141002410927.
12. Galichon, Alfred. Optimal Transport Methods in Economics. Princeton University Press, 2016.
13. Rachev, Svetlozar T., and Ludger Rüschendorf. Mass Transportation Problems: Volume I: Theory. Vol. 1. Springer, 1998.
14. Santambrogio, Filippo. Optimal Transport for Applied Mathematicians. Birkhäuser Basel, 2016. In particular chapter 6, section 4.2.
15. Aurenhammer, Franz (1987), "Power diagrams: properties, algorithms and applications", SIAM Journal on Computing, 16 (1): 78–96, doi:10.1137/0216006, MR 0873251.
16. Peyré, Gabriel and Marco Cuturi (2019), "Computational Optimal Transport: With Applications to Data Science", Foundations and Trends in Machine Learning: Vol. 11: No. 5-6, pp 355–607. DOI: 10.1561/2200000073.

17. Haker, Steven; Zhu, Lei; Tannenbaum, Allen; Angenent, Sigurd (1 December 2004). "Optimal Mass Transport for Registration and Warping". International Journal of Computer Vision. 60 (3): 225–240. CiteSeerX 10.1.1.59.4082. doi:10.1023/B:VISI.0000036836.66311.97. ISSN 0920-5691. S2CID 13261370.

18. Glimm, T.; Oliker, V. (1 September 2003). "Optical Design of Single Reflector Systems and the Monge–Kantorovich Mass Transfer Problem". Journal of Mathematical Sciences. 117 (3): 4096–4108. doi:10.1023/A:1024856201493. ISSN 1072-3374. S2CID 8301248.

19. Kasim, Muhammad Firmansyah; Ceurvorst, Luke; Ratan, Naren; Sadler, James; Chen, Nicholas; Sävert, Alexander; Trines, Raoul; Bingham, Robert; Burrows, Philip N. (16 February 2017). "Quantitative shadowgraphy and proton radiography for large intensity modulations". Physical Review E. 95 (2): 023306. arXiv:1607.04179. Bibcode:2017PhRvE.95b3306K. doi:10.1103/PhysRevE.95.023306. PMID 28297858. S2CID 13326345.

20. Metivier, Ludovic (24 February 2016). "Measuring the misfit between seismograms using an optimal transport distance: application to full waveform inversion". Geophysical Journal International. 205 (1): 345–377. Bibcode:2016GeoJI.205.345M. doi:10.1093/gji/ggw014.