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Mathematical Model of Control of a Hydrodynamic Object with Distributed Parameters (On the Example of Oil Fields)

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Annotation:

The article presents a mathematical model applied optimal control of the hydrodynamic facility with distributed parameters. The theorem on the number of switching pumps flooding in the translation object with lumped parameters from the initial to the final state.

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Introduction

The control of a large class of objects, such as oil and gas fields, as well as deposits of the mining industries, should be carried out taking into account the distribution of temperatures, pressure, flow rates, and other technological parameters, i.e., as a multi-connected control system [1,2].

To build systems for optimal control of hydrodynamic objects, it is necessary to have communication operators for the field of distributed parameters. Based on the hydrodynamic regularities of the processes, these operators can be reduced to the equations of mathematical physics [3,4].

The set of mathematical relationships between the external and internal parameters of the system, which characterize the structure of the system and its functioning, forms a model of the system.

Main part

The “reservoir-well” system, as a complex technical system, functions with some external environment, the state and properties of which at each moment of time are characterized by numerical values of a set of parameters

$\vec{\alpha} = (\alpha_1, \alpha_2, ..., \alpha_n).$

The state and properties of the system itself at each moment of time are characterized by the numerical values of a set of internal parameters

$\vec{\beta}=(\beta_1,\beta_2,...,\beta_n)$.

On the state of the system, in addition to internal and external parameters, parameters have a decisive influence $\vec{\xi}=(\xi_1,\xi_2,...,\xi_n)$, characterizing the influence of numerical values obtained with certain inaccuracies or "noise", as well as control parameters $\vec{x}=(x_1,x_2,...,x_\kappa)$.

The efficiency of the "reservoir-well" system model, i.e. the degree of its suitability for determining the characteristics of the reservoir (permeability, thickness, porosity, viscosity, compressibility, pressure, temperature, gas, oil, water, etc.) is estimated by the numerical values of the vector objective function:

$\vec{F}\ (\vec{\alpha},\vec{\beta},\vec{\xi},\vec{x})=0,$ where $\vec{F}=(F_1,F_2,...,F_\nu)$.

Many factors determine the process of gas (oil) movement in the reservoir with limited possibility of their direct measurement: the large inertia of the processes occurring in the reservoir, leading to the fact that as a result of one or another impact on the reservoir, it can be assessed after a long period of time (advancement of plantar and contour waters, various injections, impact on wells, etc.); change in reservoir properties depending on the operating mode and as the energy of gas (oil) reserves is depleted.

These features of the functioning of the "reservoir-well" system put forward special requirements for the methods of mathematical modeling of the object.

Let Ω - limited area n - dimensional space R^n, $\partial\Omega$ – smooth area border Ω, consisting of outer contour Γ_0, and internal contours $\Gamma_1,\Gamma_2,...,\Gamma_n$ where source functions can be specified. Object state $u(x,y)$ is determined by a second-order partial differential equation of elliptic type [5]:

$$\frac{\partial}{\partial x}\left[K(x,y)\frac{\partial u}{\partial x}\right]+\frac{\partial}{\partial y}\left[K(x,y)\frac{\partial u}{\partial y}\right]-\delta(u)u(x,y)=0,\ \ (x.y)\in\Omega\ \ (1)$$

under boundary conditions

$$\alpha\frac{\partial u}{\partial n}+\beta[u(x,y)+f(x,y)]=0 \text{ on the } \Gamma_0,\ (2)$$

where $\delta(u)=\begin{cases}1, & (x,y)\in\Gamma_e, e=1,2,...,n;\\ 0, & (x,y)\notin\Gamma_e.\end{cases}$

To solve the boundary value problem (1), (2), finite differences of numerical analysis are used [6].

Cyclic reservoir stimulation can be considered as a system control process with lumped parameters, where the injection pressure is taken as the control input, and the quality criterion is the achievement of the maximum amount of injected water [3]. The mathematical model of the process is a second-order differential equation obtained as a result of identification according to the statistical data of water injection [7]:

$$\ddot{Q}+T_1\dot{Q}+T_2Q=ap(t);$$

Here $Q=Q(t)$ - consumption of injected water; $p(t)$ - discharge pressure; T_1 = 0,0992; T_2 = 0,0159; a = 0,0089.

It follows from Darcy's law that the criterion for achieving the maximum amount of injected water corresponds to the minimization of the following integral relation:

$$I = -\int_{t_0}^{t^*} \frac{2\pi\kappa h(p - p_{пл})}{\mu l u \frac{r}{R}} dt.$$

Assuming parameters $\kappa, h, \mu, r, R, R_{пл}$ constant for the entire time of consideration of the process, we have an equivalent relation:

$$I = -\int_{t_0}^{t^*} p(t)dt.$$

Let us introduce the notation $x_1(t) = Q(t),\ x_2(t) = \dot{x}_1,\ u(t) = p(t),\ u^* = P_{\max}$, where $P_{\max}$ - maximum possible delivery pressure.

Suppose in the phase space X given two points $x_1^0,\ x_2^0$. For all admissible piecewise continuous controls $u(t), t_0 \le t \le t^*$ corresponds to the trajectory $\bar{x}(t)$ systems

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -T_1 x_2 - T x_1 + au, \end{cases}$$

$$x_1(t_0) = x_1^0; x_2(t_0) = x_2^0; 0 \le u(t) \le u^*, t_0 \le t \le t^*.$$

Let us introduce the function H, dependent on variables x_1, x_2, u and auxiliary variables ψ_1, ψ_2:

$$H(\psi, x, u) = \psi_1(t)x_2(t) + \psi_2(t)[-T_1 x_2(t) - T x_1(t) + au(t)] + u.$$

With this function H we write the following system of differential equations for auxiliary variables, which is conjugate to system (3):

$$\dot{\psi}_i = -\frac{\partial H(\psi, x, u)}{\partial x_1}, i = 1,2. \quad (4)$$

Theorem. If the process $(u(t),\ x(t)),\ t_0 \le t \le t^*$, transferring object from the initial state x_0 to final state x_1 is optimal, then there exists a non-trivial solution

$$\psi(t) = (\psi_1(t), \psi_2(t)), t_0 \le t \le t^*$$

system (4), which satisfies the condition of the maximum principle

$$H(\psi(t), x(t), u(t)) = \max H(\psi(t), x(t), \upsilon), \upsilon \in [0, u^*].$$

Substantiation We write system (4) in the extended form

$$\begin{cases} \dot{\psi}_1 = \psi_2 T_1, \\ \dot{\psi}_2 = \psi_2 T - \psi_1. \end{cases} \quad (5)$$

Differentiating the second equation of system (5) and substituting the values ψ_1 from the first equation, we have:

$$\ddot{\psi}_2 - T\dot{\psi}_2 + T_1\psi_2 = 0. \quad (6)$$

By the switching number theorem, for each non-trivial solution ψ_2 equations (6) uniquely determines the admissible control $u(t)$, which is a condition for the maximum of the Hamilton function, and $u(t)$ - piecewise constant function and its values are the ends of the segment $[0,u^*]$ [8].

Equation (6) has a solution

$\psi_2 = Ce^{\lambda_1 t} + C_2 e^{\lambda_2 t}$, (7)

where $\lambda_{1,2} = \dfrac{-T \pm \sqrt{T^2 - 4T_1}}{2}$ at $T^2 - 4T_1 \neq 0, \psi_2 = (C_1 + C_2)e^{-\frac{T}{2}t}$ at $T^2 - 4T_1 = 0$.

If T, T_1 - valid, then $T^2 - 4T_1 < 0$ roots λ_1, λ_2 will be complex. In this case, formula (7) is more convenient to represent in the form $\psi_2 = e^{\delta t}(A\cos\omega t + B\sin\omega t)$, where $\delta = -\dfrac{T}{2}, \omega = \dfrac{\sqrt{4T - T_1^2}}{2}$ are called constant damping and natural circular frequency c, respectively. This is the frequency if T little in relation to T_1 differs very little from the frequency T_1, which the movement would have in the absence of resistance. As the amplitude one has to consider the function of $t, e^{\delta t}$, which decreases with time, i.e. we are dealing with a damped oscillation. During one oscillation, i.e. half-cycle

$$\frac{D}{2} = \frac{2\pi}{\sqrt{4T - T_1^2}}$$

original amplitude gets a multiplier $e^{\delta \frac{2\pi}{\sqrt{4T - T_1^2}}}$. The logarithm of this expression has the form $e^{\delta \frac{2\pi}{\sqrt{4T - T_1^2}}}$. and is called the logarithmic decrement of oscillation.

The maximum condition for the Hamilton function uniquely determines the admissible control $u(t)$, at that, $u(t)$, piecewise constant function and its values are the ends of the child $[o, u^*]$:

$$\max H(t) = \psi_1(t)x_2(t) + \psi_2(t)[-T_1 x_2(t) - Tx_1(t) + au(t)] + u,\ u \in [0, \mathrm{u}^*],$$

which are achieved with the following control values

$$u(t) = \begin{cases} u^*, & if\, \psi_2 a + 1 > 0, \\ 0, & if\, \psi_2 a + 1 < 0. \end{cases}$$

Conclusion

So, given that the frequency value is known, it is possible to determine the moments of switching of the water flooding pumps. By the switching number theorem, for every nontrivial solution $\psi_2(t)$ constants C_1, C_2 cannot both be equal to zero. For various values C_1, C_2 satisfying $a\sqrt{C_1^2 + C_2^2}$ the amplitude of the sinusoid changes, the points of its intersection with the axis t remain unchanged. Therefore, the optimal control is a piecewise constant function (the discontinuities correspond to the moment of switching the water flood pumps), which takes the maximum possible and zero values. The given applied research on the optimal control of a deterministic control object can be successfully applied in practical problems of water flooding of mineral deposits.

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