

Mathematical Modeling of Learning Processes Based on the Theory of Control

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Annotation:

The learning process is considering as object of optimal control, for process controlling offered sequence of innovative approaches. Given a functional structure didactic system "teacher-reader" and on this bases created cyclic structure of calculation experience. The obtained results are graphically displayed.

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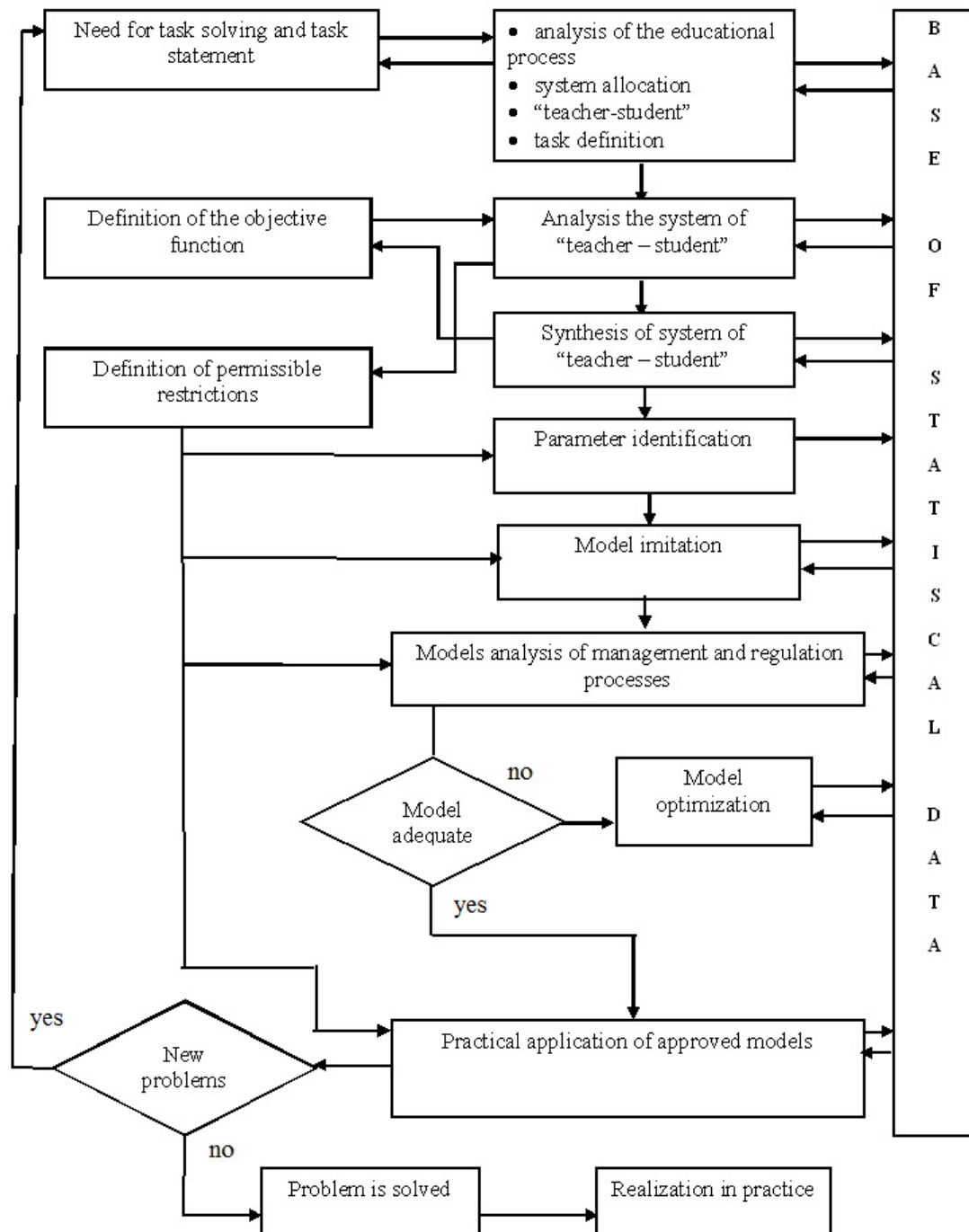
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Introduction

The management problems of didactic systems and the methods of mathematical modeling of learning processes have been studied in numerous works by L.P. Leontyev, O.G. Gokhman, R.V. Mayer, N.F. Talyzin et al. [1-3]. In these works, some aspects of the optimal management of educational processes at the university are considered. In particular, the development of an optimal curriculum, measurement of educational information, communication model of the volume of the stated and learned material, quantization of educational material, the principle of feedback and other statements and solving applied control problems (program and optimal) for studying learning processes begins with the creation of the management structure of the studied processes. The functioning of the created structure of the open control system, as a non-deterministic object, has no regularity of development. At the same time, methods of simulation modeling are applied using the principle of the system approach of control theory [4]. Therefore, the development of mathematical models for solving problems of analysis and synthesis of social objects of management, as a learning process, is highly relevant.

Main part

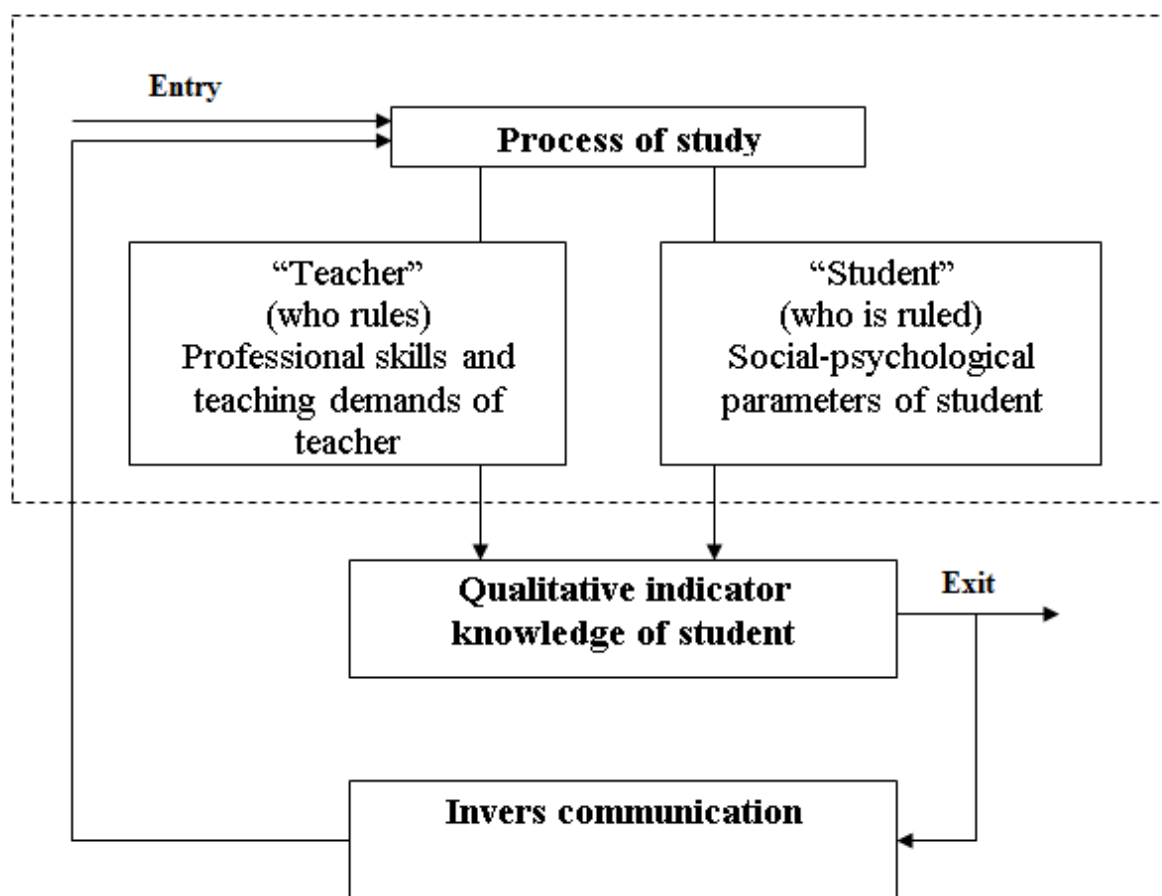
This article is devoted to theoretical studies with a position of a systematic approach to learning processes with the creation of a structure of a sequence of actions (Pic.1). The functional structure of the learning process as a didactic system “teacher-student” has been created. The theoretical justification created the structure of functioning and proposed a mathematical model of functioning with the aim of optimal control and analysis of the state of the system as a whole.



Pic.1 Structural subdivision is a novelty in the podcode and didactic system "Teacher-student."

Learning processes in secondary schools as an object of social management consists of pupils (receiver of information) and teachers (source of information) of the subject area. The functional structure of the educational process in school education consists of several socio-psychological parts (Pic. 2). The exchange of information and the assessment of their interaction between individual parts is one of the main goals of this work.

The use of a systematic approach to the theory of cybernetics allows us to obtain a control structure with a hierarchy of development in space and time. Since the object under study is an open control system, the practical application of the developed models in non-deterministic objects is relevant both from the theoretical and applied side.



Pic.2. The functional structure of the learning process in the teacher-student didactic system.

Analysis of the situation is reduced to the study of the learning process. At this stage, along with the system under study, the situation of educational processes outside it is analyzed, i.e. in an environment in which the system under consideration is or should be included. It is necessary to get a general idea of how to solve a problem (problem), since they determine the choice of the most realistic goals. The result of the analysis of the learning process (situation) is a list of problems and alternatives to solve them, which are investigated at the stage of the system being developed.

Based on the results of the analysis of the learning process, the goals of the tasks are formulated. At the stage of analysis of the system, its elements are mentally divided, with the properties of each being singled out, connections with each other and with the external environment as an information exchange vertically and horizontally as a hierarchical multiply connected system. The purpose of the analysis is to separate the essential properties and relations in the system from the inessential, to move on to a deeper study of it. For

this, the analyzed properties and relations are considered in the relationship, i.e. analysis is associated with the synthesis and provides the didactic unity of these methods of knowledge.

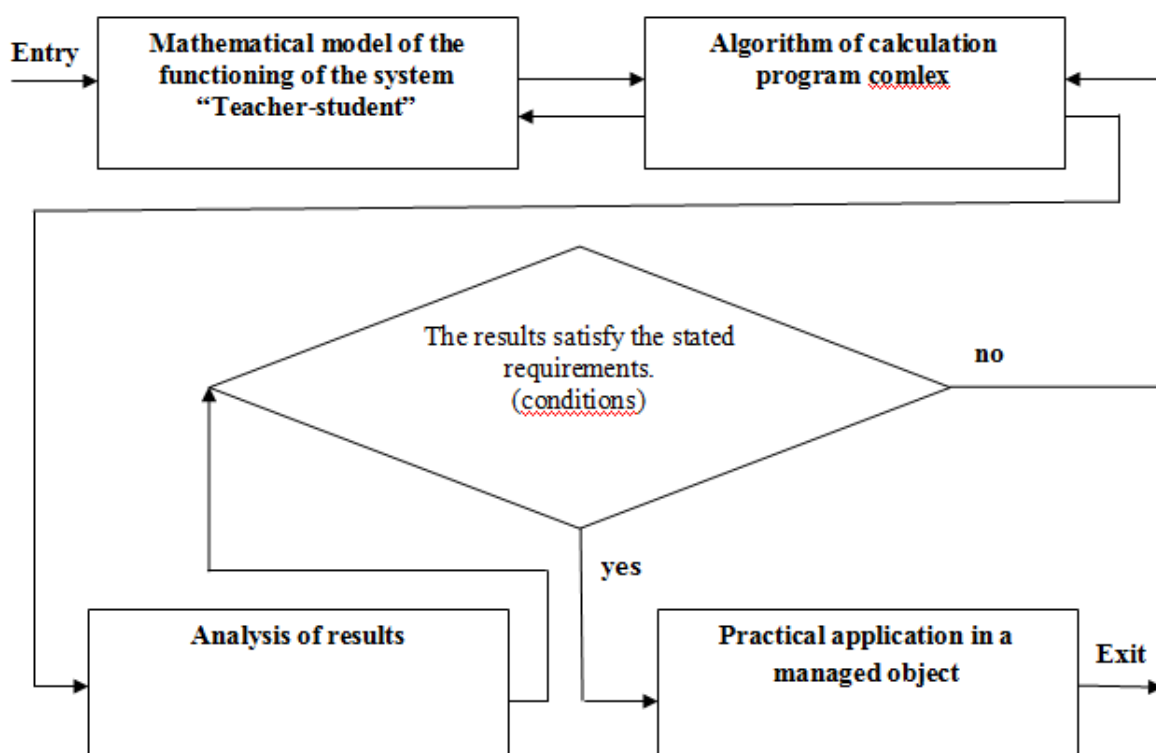
Identification - an accurate description of the structure and parameters of the system model. For identification, modern mathematical-statistical methods are used, such as the method of principal components, the method of heuristic self-organization, regression analysis, etc.

After the mathematical model of the system has been built, the state of the system is checked using simulation. And also with the help of the obtained results of imitation, the adequacy of the model of the system under study is checked. If the required level of reality is not achieved, then the solution of the problems of analysis and synthesis is repeated. The computational experiment continues until a satisfactory quantitative result is obtained. With obtaining the required results of analysis and synthesis, practical application of the developed models is carried out (pic. 3).

The system under study is a dynamic system, i.e. in it, all changes occur in time, and models that reflect these changes are dynamic models of the system.

In mathematical modeling of the learning process, its specific implementation is described as a correspondence between the elements of the set of possible “values” and the elements of an ordered set of “moments of time”, i.e. as a display.

With the help of these concepts a mathematical model of the system is built [5].



Pic.3. Periodic structure of calculating experiment

Consider a way out $y(t)$ “teacher-student” systems as her response to managed $U(t)$ and unmanaged $v(t)$ out $x(t) = \{U(t), v(t)\}$ The model can be expressed as a combination of two processes: $X^T = \{x(t)\}$ and $Y(t) = \{y(t)\}, t \in T$

The state of the system as a certain (internal) characteristic of the system, the value of which currently determines the current value of the output value (knowledge, teachings). The state can be considered as a kind of information repository (knowledge) necessary to predict the influence of the present on the future. This means the existence of such a mapping. $\eta : Z \cdot T \rightarrow Y$ what $y(t) = \eta(t, Z(t)), t \in T$

An explicit dependence on introduced to take into account the possibility of a change in the dependence of the output on the state over time. This mapping is called the output mapping.

To build a model, we describe the relationship between the input and the state, i.e. introduce parametric mapping family $\mu_{\tau} : Z \cdot X(\bullet) \rightarrow Z$, set for all parameter values $t = T$ and $\tau \leq t_0$. This means that the state at any time $t > \tau$ uniquely determined by the state Z_{τ} in the moment τ and segment implementation of the entrance $x(\bullet)$ from τ till t $Z(t) = \mu_{\mu}(Z_{\tau}, x(t)) = \sigma(t, \tau, Z_{\tau}, x(\bullet))$. This mapping is called transitive mapping.

The study of the learning process as a cybernetic research system (theoretically) solves the problems of developing a mathematical model of the control object, identifying the special values of feedback in the system, emphasizing optimal control and system synthesis, transmitting and processing information and its quantitative description, developing a computational experiment .

Optimum software control of the learning process using the Lagrange-Pontryagin method for continuous learning processes is given in [5-7].

Requires optimal way to get from the point $(Y_0, 0)$ to the point (Y_1, T) , where $Y_1 \in [Y_{\min}, Y_{\max}]$. In the first approximation, we take the straight line connecting the starting and ending points

$$Y^0(t) = Y_0 + \frac{Y(T) - Y_0}{T} t, t \in [0, T] \tag{1}$$

To calculate the optimal programmed control and optimal trajectory, the following formulas were proposed in [7-9]:

$$u_0^*(t) = \frac{Y^0(t)(kp + 1)}{k_0p + 1}, \tag{2}$$

$$Y^*(u_0^*, t) = Y_0 e^{-kt} + e^{-kt} \int k_0 u_0^*(t) e^{kt} dt \tag{3}$$

where is the additional variable

$$p(u_0, t) = \frac{Y_0 - u_0(0)}{k_0 u_0(0) - k Y_0} e^{kt} - \frac{1}{k} + \frac{1}{k} e^{kt} \tag{4}$$

In the same way, optimal software control is calculated for self-learning. u_2^* and optimal trajectory $Y^*(u_2^*, t)$:

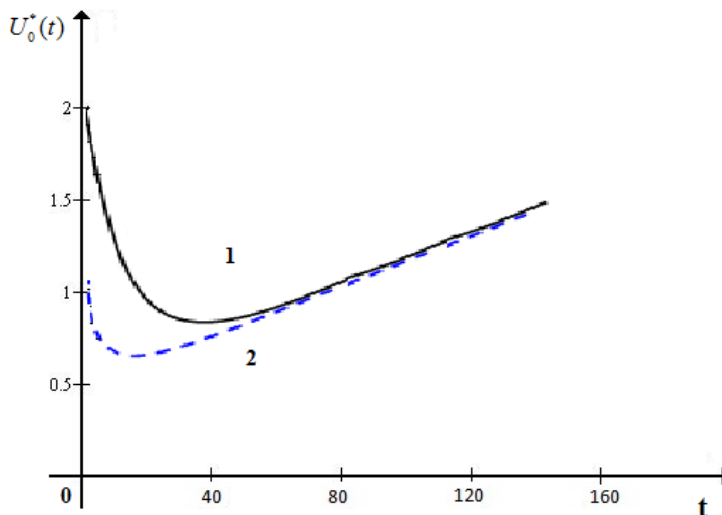
$$u_2^*(t) = \frac{Y_0(t)(k_2p + 1)}{k_2p + 1}, \tag{5}$$

$$Y^*(u_2^*, t) = Y_0 e^{-kt} + e^{-kt} \int k_2 u_2^*(t) e^{kt} dt \tag{6}$$

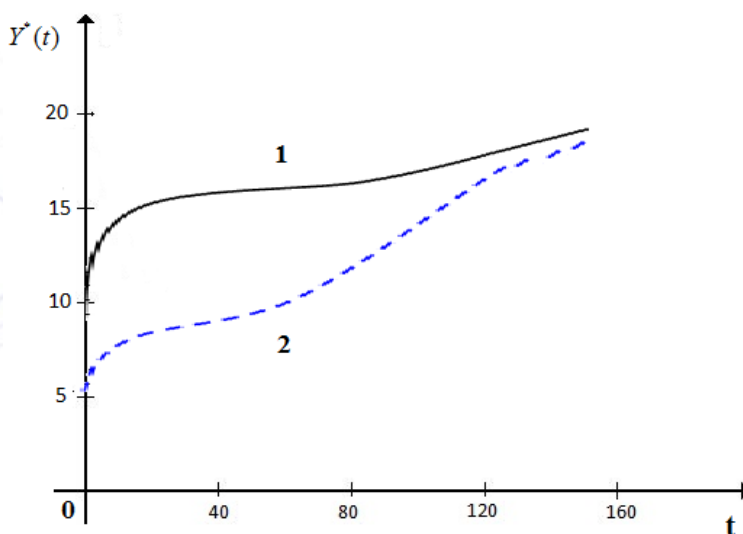
where we take as an additional variable

$$p(u_2, t) = \frac{Y_0 - u_2(0)}{k_2 u_2(0) - k Y_0} e^{kt} - \frac{1}{k} + \frac{1}{k} e^{kt} \tag{7}$$

As specific process parameters: $T = 160, Y_1 = 22, k = 0,33, k_0 = 0,9$, with a total load given by the teacher - 30 hours. Then the graphs of optimal programmed control and optimal trajectory for different initial conditions are presented in Pic. 4, curve 1 – $Y_0 = 8, u_0(0) = 2$,



Pic. 4. Graph of optimal program control in time.



Pic. 5. Graph of the optimal trajectory for program control in of time.

curve 2 – $Y_0 = 4, u_0(0) = 1$, On the picture 5, curve 1 – $Y_0 = 8, u_0(0) = 2$, curve 2 – $Y_0 = 4, u_0(0) = 1$.

The training load of the student with this program management is determined by the area of the integrand curve in Fig. 4 and is equal to 120 hours (at a given teacher at 30 hours), therefore, for a real learning process, feedback control is also necessary.

We are talking about deterministic systems with “continuous time”, i.e. about systems whose evolution is described by differential equations. But of no less importance are systems with “discrete time”. Their role is determined not only by the fact that when constructing computational procedures we always carry out discretization of the variables of the problem posed - we replace differential equations with finite-difference ones. These are primarily multi-step decision making tasks. Almost always the development of socio-

economic systems is described by finite-difference equations. The discretization step is determined by the cycle of the educational process (month, quarter, semester, academic year).

Conclusion

The proposed mathematical model of learning based on the control theory can be useful, first of all, by the manager of the teaching process and teachers. On the basis of the coefficients of learning and forgetting, determined with the help of special tests, it is possible to predict in some approximation the level of current knowledge of both the individual student and the group of students (or the flow of students). Thus, the learning process can be controlled more accurately compared to the traditional approach.

The use of these solutions in practice will improve the quality of education and preserve the knowledge of graduates of educational institutions in the long-term plan with a minimum load of teaching staff.

Based on the proposed mathematical models of optimal control, you can create an automated control system that allows you to optimally plan the pedagogical process.

The presented model of optimal control can be used by managers of different levels of education management to solve applied problems of analysis, management and regulation of the educational process.

The theory of mathematical modeling can be used in advanced training courses for engineering and technical workers of industrial enterprises by industry.

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