Newton-Cotess Type Quadrature Formulas

Ziyakulova Shahnoza Abdirasulovna

Teacher, Department of Applied mathematics and informatics, Termez state university

Abstract:

This paper discusses the problem of approximate calculation formulas for integrals. The integrals are illustrated with examples, for which a C ++ program is presented and the solutions obtained are compared.

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INTRODUCTION

The exact integral of a given function y = f (x)

$$\int_{a}^{b} y\,dx$$

required to calculate.

Selecting the constant $h = \frac{b-a}{n}$ steps [a, b] points of equal distance from the intersection

$$x_0 = a, \ \ x_i = x_0 + ih \ \ \ (i = 1,2,\dots,n-1), x_n = b$$

We divide n into equal parts and the values of the function f (x) at these points

$$y_i = f(x_i), \ \ i = 0,1,2,\dots,N$$

let it be known.

Main Part

Substituting the function y (x) under the integral with the Lagrangian interpolation multiplication $L_n(x)$, we obtain the following approximate quadratic formula

$$\int_{x_0}^{x_n} y\,dx = \sum_{i=0}^{n} A_i\, y_i \qquad\qquad (1)$$

where A_i is a constant coefficient. We derive exactly for the A_i coefficients in formula (1). As you know,

$$L_n(x) = \sum_{i=0}^{n} P_i(x) y_i \qquad (2)$$

here

$$P_i(x) = \frac{(x-x_0)(x-x_1)\dots(x-x_{i-1})(x-x_i)(x-x_{i+1})\dots(x-x_n)}{(x_i-x_0)(x_i-x_1)\dots(x_i-x_{i-1})(x_i-x_{i+1})\dots(x_i-x_n)} \qquad (3)$$

Enter this notation $P_i(x) = \frac{W(x)}{(x-x_i)W'(x_i)}$

$$q = \frac{x-x_0}{h} \qquad (4)$$

and

$$q^{[n+1]} = q(q-1)\dots(q-n) \qquad (5)$$

taking $\omega(x) = (x - x_0)(x - x_1)\dots(x - x_n)$ and

$$\omega'(x_i) = (x_i - x_0)(x_i - x_1)\dots(x_i - x_{i-1})(x_i - x_{i+1})\dots(x_i - x_n)$$

we rewrite the expressions, taking into account the following

$$x - x_0 = hq, \quad x_{i+1} = x_i, \qquad x_i = x_0 + ih$$
$$x - x_1 = x - (x_0 + h) = x - x_0 - h = hq - h = h(q - 1),$$
$$x_i - x_1 = x_0 + ih - (x_0 + ih) = x_0 + ih - x_0 - h = h(i - 1)$$

In this case

$$\omega(x) = h^{n+1}q(q-1)\dots(q-n) = h^{n+1}q^{[n+1]}$$
$$\omega'^{(x)} = h^n i(i-1)\cdots 1 \cdot (-1)\cdot(-2)\cdots[-(n-i)] =$$
$$(-1)^{n-i}h^n i!\,(n-i)!$$

will be. Substituting these values of $\omega(x)$ and $\omega'(x)$ into (2), we obtain

$$L_n(x) = \sum_{i=0}^{N} \frac{\omega(x)}{(x-x_i)\omega'(x_i)} y_i = \sum_{i=0}^{N} \frac{(-1)^{n-i}}{i!\,(n-i)!}\cdot\frac{q^{[n+1]}}{q-i} y_i \qquad (6)$$

for $L_n(x)$.

If we replace the function y (x) in integral (1) with Lagrange interpolation increases $L_n(x) - (6)$, as well as the integration variable $q = \frac{x_n - x_0}{h}$ in the exact integral with $dq = \frac{dx}{x}$, the limits of the exact integral are defined as follows:

$$x_n = x_0 + nh$$

$$x = x_n, \quad q = \frac{x_n - x_0}{h} = \frac{x_0 + nh - x_0}{h} = \frac{nh}{h} = n$$

For the coefficients A_i, $x = x_0$ and $q = \frac{x_0 - x_0}{h} = \frac{0}{h} = 0$ we obtain the following formula

$$A_i = h \frac{(-1)^{n-i}}{i!\,(n-i)!} \int_0^n \frac{q^{[n+1]}}{q-i}\,dq, \quad i = 0,1,2,\dots,n$$

A_i coefficients are rewritten. Since $h = \frac{b-a}{h}$, usually it is taken as $\qquad A_i = (b-a)H_i$, in this

$$H_i = \frac{1}{n}\frac{(-1)^{n-1}}{i!\,(n-i)!} \int_0^n \frac{q^{[n+1]}}{q-i}\,dq \quad (i = 0,1,\dots,n) \qquad (7)$$

Formula (7) is called the Cotess coefficients. The squaring formula (1) in this case takes the following form.

$$\int_a^b y\,dx = (b-a)\sum_{i=0}^{n} H_i y_i \qquad (8)$$

in this

$h = \frac{b-a}{n}$ and $y_i = f_i(a + ih), (i = 0,1,2,\dots,n)$

The following equations are valid:

$$1) \quad \sum_{i=0}^{n} H_i = 1 \; ; \quad 2) \; H_i = H_{n-i} \quad .$$

Using formula (8) it is possible to formulate integral calculation formulas.

We use formula (8) at $n = 1$. In this case $i = 0; 1$. We find that

when $i = 0$, $H_o = -\int_o^1 \frac{q(q-1)}{q}\,dq = \frac{1}{2}$ and

when $i = \emptyset$, $H_1 = \int_0^1 q\,dq = \frac{1}{3}$.

Now from the formula

$$\int_{x_o}^{x_1} y\,dx = (x_1 - x_0)\sum_{i=0}^{1} H_i y_i$$

we form the trapezoidal formula

$$\int_{x_0}^{x_1} y\,dx = \frac{h}{2}(y_{0+}y_1) \;.$$

This is the residual value of the formula

$$R = \int_{x_0}^{x_1} y\,dx - \frac{h}{2}(y_0 + y_1) = -\frac{h^3}{12}y''(\xi), \xi \epsilon(x_0, x_1)$$

To calculate the integral $\int_a^b y\,dx$, we divide the integration integral $[a, b]$ into n equal parts:

$$[x_0, x_1], [x_1, x_2], \dots, [x_{n-1}, x_n]$$

Now we use the trapezoidal formula for each of them.

Assuming $h = \frac{b-a}{n}$, if we denote by $y_i = f(x_i)(i = 0,1,\dots,n)$

the values of the function under the integral at points x_i, we obtain the formula

$$\int_a^b y\,dx = h[\frac{y_0}{2} + y_1 + y_2 + \cdots + y_{n-1} + \frac{y_n}{2}] \qquad (9)$$

This is the residual value of the formula

$$R = \int_{x_0}^{x_n} y\,dx = \frac{h}{2}\sum_{i=1}^{n}(y_{i-1} + y_i) = \sum_{i-1}^{n}[\int_{xi-1}^{xi} y\,dx - \frac{h}{2}(y_{i-n} + y_i)] =$$

$$= -\frac{h}{12}\sum_{i-1}^{n} y''\left(\varsigma_i\right) \qquad (10)$$

here $\xi_i \in (x_{i-1}, x_i)$. (9) is a generalized trapezoidal formula.

Now from formula (7) at $n-2$, we obtain the following for $i = \emptyset, 1, 2$

$$H_0 = \frac{h}{2} \cdot \frac{h}{2}\int_0^2 (q-1)(q-2)\,dq = \frac{1}{4}\left(\frac{8}{3} - 6 + 4\right) = \frac{1}{6},$$

$$H_1 = -\frac{1}{2}\int_0^2 q(q-2)\,dq = \frac{2}{3},$$

$$H_2 = \frac{1}{2} \cdot \frac{1}{2}\int_0^2 q(q-1)\,dq = \frac{1}{6}.$$

In turn, since $x_2 - x_0 = 2h$, we have

$$\int_{x_0}^{x_2} y\,dx - \frac{h}{3}(y_0 + 4y_1 + y_2) \qquad (11)$$

Formula (11) is called the Simpson formula, and the residual limit of the Simpson formula is equal to

$$R = \int_{x_0}^{x_2} y\,dx - \frac{h}{3}(y_0 + 4y_1 + y_2) = -\frac{h^5}{90}y'\left(\xi\right), \xi\epsilon(x_0, x_2)$$

Now, in deriving the generalized Simpson formula, let $n = 2m$ be an even number and $y_i = f(x_i), i = 0,1,2,\ldots,n$ the values of the function $y = f(x)$ in $h = \frac{b-a}{n} = \frac{b-a}{2m}$ steps at equal points $a = x_0, x_1, \ldots, x_n = b$

If we use the Simpson formula for each binary section, that is for

$$[x_0, x_2], [x_2, x_4], \ldots, [x_{2m-2}, x_{2m}],$$

the length of each is $2\,h$. We have a generalized form of Simpson's formula.

$$\int_a^b y\,dx = \frac{h}{3}[(y_0 + y_{2m}) + 4(y_1 + y_3 + \cdots + y_{2m-1}) + 2.(y_2 + y_4 + \cdots + y_{2m-2})]$$

If we write this formula differently,

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}(y_0 + 4y_1 + y_2) + \frac{h}{3}(y_2 + 4y_3 + y_4) + \cdots + \frac{h}{3}(y_{2m-2} + 4y_{2m-1} + y_{2m})$$

$$\int_{x_0}^{x_n} y\,dx = \int_{x_0}^{x_2} y\,dx + \int_{x_2}^{x_4} y\,dx + \cdots + \int_{x_{2m-2}}^{x_{2m}} y\,dx \qquad (12)$$

This is the residual value of the formula

$$R = -\frac{mh^5}{90}y'^v(\xi) = -\frac{(b-a)h}{180}y'^v(\xi)$$

Here $\xi \in [a,b]$.

For example. Calculate $\int_1^2 (x^2+1)dx$ in the trapezoid and Simpson formula.

$\int_1^2 (x^2+1)dx = \left(\frac{x^3}{3}+x\right)\Big|_1^2 = \frac{8}{3}+2-\frac{1}{3}-1 = \frac{10}{3}$. Now we calculate using the trapezoidal formula.

Let n=4.

```
1    #include <iostream>
2    #include <math.h>
3
4    using namespace std;
5
6    int main()
7    { float a,b,h; float s=0,p=1;
8      int n;
9      cout << "Enter a = " ;cin >> a;
10     cout << "Enter b = " ;cin >> b;
11     cout << "Enter n = " ;cin >> n;
12     h=(b-a)/n; cout << "h=" << h <<endl;
13     s=(a*a+1+b*b+1)/2;
14     for (float i=a+h; i<b; i=i+h)
15     {
16         s=s+i*i+1;
17
18     cout << "i="<< i << "s= "<< s << endl;
19     }
20     p=s*h;
21     cout<< "p=" <<p << endl;
22         return 0;
23     }
24
```

```
Enter a = 1
Enter b = 2
Enter n = 4
h=0.25
i=1.25s= 6.0625
i=1.5s= 9.3125
i=1.75s= 13.375
p=3.34375

Process returned 0 (0x0)   execution time : 8.503 s
Press any key to continue.
```

<div align="center">Calculation using the Simpson formula</div>

```
main.cpp  X
1    #include <iostream>
2    #include <math.h>
3    using namespace std;
4    int main()
5    {float a,b,h,s=0,f;
6      int n;
7        cout << "a=";cin>>a;
8        cout << "b=";cin>>b;
9        cout << "n=";cin>>n;
10       h=(b-a)/n;
11       s=(a*a+1)+(b*b+1);
12       cout<<"h="<<h<<endl;
13       for (int i=1; i<n;  i++)
14       {float x=a+i*h;f=x*x+1;
15        if (i%2==1) s+=(4*f);else s+=(2*f);}
16        s*=h/3;
17
18       cout<<"s="<<s<<endl;
19
20       return 0;
21    }
22
```

```
C:\Users\User\Desktop\misol\simpson22\bin\Debug\simpson22.exe
a=1
b=2
n=4
h=0.25
s=3.33333

Process returned 0 (0x0)   execution time : 2.991 s
Press any key to continue.
```

If we compare the results, we can see that they are almost similar.

Conclusion

You need to know the initial tables to solve examples of integrals. It will be difficult to find the initial function of all the functions under the integral. Therefore, approximate methods are needed to calculate integrals. Newton-Cotess-type quadrature formulas have a high degree of accuracy in the approximate calculation of integrals. It is possible to form a trapezoidal formula at n = 1, Simpson's formula at n = 2 and other formulas.

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